# Parameter Identification of One Class of Nonlinear Systems Using Hammerstein Model with Feedback

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***Abstract*—The parameter identification problem of the nonlinear systems functioning with positive feedback using a continuous Hammerstein model with unit feedback is considered. The method of parameter identification based on the observation of the system's input and output variables in the steady state at the input harmonic influences is proposed. The solution to the parameter identification problem is carried out by the method of least squares. The parameter identification algorithm is investigated by means of both theoretical analysis and computer modeling.**

***Keywords*—nonlinear system, block-oriented model, identification, parameter, feedback**

## I. Introduction

Nonlinear systems that function with feedback are widespread in industrial processes, particularly in metallurgical, mining, chemical, pulp-and-paper industry, ecology, etc. One class of such systems is defined by the fact that a part of the initial material remained unprocessed up to the required condition, when passing through to the working part of the object, returns to the entry of this object for reprocessing. Such closed-loop systems with positive feedback are characterized by maximum raw material utilization and comparatively high efficiency [1].

In order to determine the regularity of current processes and for their control in nonlinear systems, block-oriented models are commonly used, consisting of different modifications of the Hammerstein and Wiener models [2] or general models, in particular, the Volterra [3] and Wiener [4] series and the Kolmogorov-Gabor [5-6] continuous and discrete polynomials.

When building models of nonlinear systems with feedback, it is necessary to consider the fact that there is some a priori information about the system. For example, for the mill of ore-dressing plant working with feedback, proceeding from their functioning conditions, there is certain information about the static characteristic of the system, which can be approximated by a polynomial function of the second degree, and system inertance, is considered in the form of linear dynamic - in particular, aperiodic elements [7]. Therefore, we can use block-oriented models for modeling such nonlinear systems.

During the construction of the system’s mathematical model by using the system identification methods it is necessary to solve different problems depending on the a priori information about the system [8]. The construction of the system’s adequate model in many respects depends on successfully solving structure and parameter identification problems.

Usually, the model structure is determined based on a priori information or depending on the physical laws of the processes that take place in the system [8]. However, the structure of the model determined this way often has high dimensions, and its application is not expedient for the solution of practical problems.

In the following years, there were several papers (e.g., [9-11]) in which different aspects of estimation of the structure of the model by data of system input-output were considered. As for the problem of parameter identification problem, it can be usually solved based on the use of experimental data obtained through experiments conducted in the system.

Due to the functioning peculiarities of nonlinear systems with feedback and mathematical difficulties at the solution of nonlinear differential equations, which describe processes in the systems, identification problems in such systems are much more complex than the same problems of open nonlinear systems.

This work uses the characteristics of the non-linear production systems with positive feedback to determine the mathematical equation describing the Hammerstein model.

In the work, the problem of parameter identification of the nonlinear systems with feedback at their representation by the Hammerstein model with unit feedback based on the observation of the system's input and output variables at the input harmonic influences is considered. The offered parameter identification method allows determining the first part of the parameters – static characteristics in the stationary state, and the second part of the parameters – dynamic characteristics in the steady state in the frequency domain by using the Fourier approximation based on the method of the least squares.

## II. Some Peculiarities of the Functioning Systems with Feedback

In complex nonlinear systems functioning with positive feedback, the steady movement at their output is reached only at certain values of the parameters of the system and under the change of the input influence within certain limits.

A system structural scheme with feedback has the following form (fig. 1):

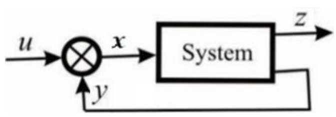


Fig. 1. A system structural scheme with feedback

The initial product $u$ feeds into the system’s input, and in the system output a new product $z$ and an unprocessed part of raw material $y$ (a recycle), which again feeds into the input,

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are produced. A total feeding implying an output value and recycled material is signed by *x*. According to their nature, none of the variables *u, x, y, z* can be negative. According to the above

$$x = u + y\,.$$

In the steady state of an open system an equation

$$x = y + z$$

is true (here of course $x=u$) and for a system with feedback and equality

$$u = z\,,$$

that follows from the balance of the materials weighting quantities, incoming and outcoming from the system in the time unit.

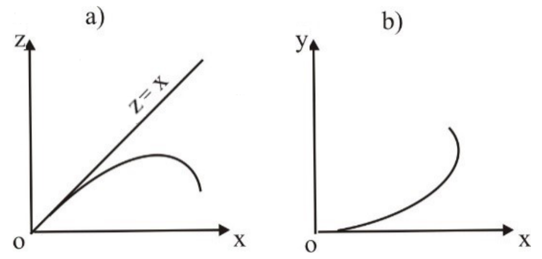


Fig. 2. Static characteristics of the open system:
a) for a direct channel,
b) for a feedback channel.

For an object with an open cycle of a characteristic $z=q(x)$ and $y=f(x)$ (fig. 2, a), b)) are essentially nonlinear and are static characteristics. The characteristic $y=f(x)$ is the addition of $z=q(x)$ up to a straight-line $y=x$.

Proceeding from the features of systems working with feedback, it is conditionally possible to pick out the direct channel and the channel of feedback. There is a transformation of the initial material to the ready product with the direct channel, and there is a movement of recyclable material in the system with the channel of feedback. In practice, the ready product is usually transferred to the following technological process cycle, and the initial and recyclable materials are measured. The knowledge of the mathematical model of the channel of feedback enables the construction of a model for the whole system.

## III. CLASSES OF MODEL AND INPUT SIGNALS

Let's consider the Hammerstein model with unit feedback for the presentation of the feedback channel of nonlinear closed-loop systems (fig. 3).

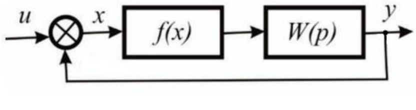


Fig. 3. The Hammerstein model with unit feedback

This model is described by the equations:

$$y(t) = W(p)\, f\left[u(t) + y(t)\right], \tag{1}$$

where *w(p)* is the transfer functions in the operator form of the model's linear dynamic parts and $p$ - designates the differentiation operation: $p \equiv d/dt$ .

On the basis of the a priori information [2], it is supposed that the nonlinear static element, which is part of block-oriented models with feedback, is described by the polynomial function of the second degree:

$$f(x) = c_1 x + c_2 x^2 \tag{2}$$

and the transfer function of the linear dynamic element has the following form:

$$W(p) = \frac{1}{Tp+1}\,, \tag{3}$$

where $c_1,\ c_2$ - constant coefficients and the free member $c_0$ is absent since there is no signal on the output of the system with feedback at the zero-input signal, and $T$ - time constant.

If we consider expressions (2) and (3) in expression (1) after several transformations, we get that the Hammerstein model is described by equations:

$$\dot{x}(t) = -\frac{1-c_1}{T}x(t) + \frac{c_2}{T}x^2(t) + \dot{u}(t) + \frac{1}{T}u(t)\,, \tag{4}$$

$$y(t) = x(t) - u(t)\,. \tag{5}$$

For solving the problem of parameter identification of nonlinear systems it is supposed that the input variable of the system $u(t)$ is a harmonic function:

$$u(t) = A\sin\omega t + B\,. \tag{6}$$

## IV. MATHEMATICAL DESCRIPTION OF OUTPUT FORCED OSCILLATIONS

In order to solve the parameter problem, it is necessary to solve differential equation (4) taking into account (5), (6), and also to consider stability conditions of the steady motion at the output of the closed-loop nonlinear systems.

For the nonlinear systems with feedback, it was obtained [2], that the implementation of the following conditions:

$$0 < c_1 < 1,\ \ \mathrm{c}_2 > 0\,, \tag{7}$$

and

$$\bar{u} < \frac{1-c_1}{2c_2}\,, \tag{8}$$

where $\bar{u}$ - a value of the input signal for some steady state, guarantees the system stability. Therefore, for the Hammerstein model it is supposed that the conditions (7) - (8) are valid.

According to the above-mentioned, for the solution of Riccati equations, corresponding to the model, it is possible to use the method of a small parameter and to search for the solution of the equations in the form of the following series:

$$x(t)=\sum_{n=1}^{\infty}\mu^n x_n(t), \qquad (9)$$

where $\mu$ is a small parameter.

If considering (6) in equation (4), we get:

$$\dot{x}(t)=-\frac{1-c_1}{T}x(t)+\frac{c_2}{T}x^2(t)+\frac{1}{T}A\sin\omega t+$$
$$+A\omega\cos\omega t+\frac{1}{T}B. \qquad (10)$$

It follows from conditions (7) and (8) that $c_2$ is a small parameter and we can assume that:

$\mu=c_2$.

The smallness of $c_2$ ensures rapid convergence of the solution (9). If in the expression (9) members of the second and higher orders small values are not taken into account we can be limited to two members:

$$x(t)=x_0(t)+c_2x_1(t). \qquad (11)$$

If we put (11) in equation (4) and equate coefficients of terms with the same degree of $c_2$ on the right and left sides of the equation, we get:

$$\dot{x}_0(t)=-\frac{1-c_1}{T}x_0(t)+\frac{1}{T}A\sin\omega t+A\cos\omega t+\frac{1}{T}B, \qquad (12)$$

$$\dot{x}_1(t)=-\frac{1-c_1}{T}x_1(t)+\frac{1}{T}x_0^2. \qquad (13)$$

Equations (12) and (13) are first-order linear ordinary differential equations. If we solve equation (12) with zero initial condition and consider the obtained solution in equation (13), then by solving it also at zero initial conditions and take into account (5), we get that the output variable of the model the steady state is represented as follows:

$$y(t)=\frac{c_1(1-c_1)^2B+c_2B^2}{(1-c_1)^3}+\frac{c_2A^2(1+\omega^2T^2)}{2(1-c_1)\left[(1-c_1)^2+\omega^2T^2\right]}-$$
$$-A\sin\omega t+\frac{A\sqrt{(1+\omega^2T^2)}}{\sqrt{(1-c_1)^2+\omega^2T^2}}\sin(\omega t-\varphi_1)+$$
$$+\frac{2c_2AB\sqrt{(1+\omega^2T^2)}}{(1-c_1)\left[(1-c_1)^2+\omega^2T^2\right]\sqrt{(1-c_1)^2+\omega^2T^2}}\sin(\omega t-\varphi_1-\varphi_2)-$$
$$-\frac{c_2A^2(1+\omega^2T^2)}{2\left[(1-c_1)^2+\omega^2T^2\right]\sqrt{(1-c_1)^2+4\omega^2T^2}}\cos(2\omega t-2\varphi_1-\varphi_3), \qquad (14)$$

where

$$\varphi_1=arctg\frac{c_1\omega T}{1-c_1+\omega^2T^2},$$

$$\varphi_2=arctg\frac{\omega T}{1-c_1},$$

$$\varphi_3=arctg\frac{2\omega T}{1-c_1}.$$

## V. PARAMETER IDENTIFICATION

A considerable quantity of scientific works, in which this problem is solved based on different approaches and methods, is devoted to the problem of parameter identification of nonlinear systems.

At the representation of nonlinear systems by block-oriented models, most of the parameter identification methods are developed for opened models (e.g., [12-15]. In the same works [16-17] offered a method of parameter identification of nonlinear systems with feedback that allows defining a part of parameters – static characteristics in the stationary state, and the second part of parameters – dynamic characteristics in a transitive mode. But when determining the dynamic characteristics, it is necessary to calculate the derivatives based on experimental data related to the acceptance of errors. It is necessary to note that there are also other approaches (e.g., [18-19]) to identification of nonlinear systems by using feedback.

The majority of block-oriented models with the feedback are nonlinear concerning the parameters and the analytical solution of the parameter identification problem is possible for some low-order models.

### A. *Identification of static parameters*

For the Hammerstein model with unit feedback, the connection between the input and output variables in the stationary state is determined by the equation:

$$(1-c_1)x-c_2x^2=u, \qquad (15)$$

where the connection between the variables *x* and *y* is defined by (1).

Static parsmeters estimates by the least squares method were previously obtained [16-17] using expressions (15):

$$\hat{c}_1=1-\frac{\left(\sum_{i=1}^{n}x_i^4\right)\left(\sum_{i=1}^{n}u_ix_i\right)+\left(\sum_{i=1}^{n}x_i^3\right)\left(\sum_{i=1}^{n}u_ix_i^2\right)}{\left(\sum_{i=1}^{n}x_i^2\right)\left(\sum_{i=1}^{n}x_i^4\right)-\left(\sum_{i=1}^{n}x_i^3\right)^2}, \qquad (16)$$

$$\hat{c}_2=\frac{\left(\sum_{i=1}^{n}x_i^2\right)\left(\sum_{i=1}^{n}u_ix_i^2\right)+\left(\sum_{i=1}^{n}x_i^3\right)\left(\sum_{i=1}^{n}u_ix_i\right)}{\left(\sum_{i=1}^{n}x_i^2\right)\left(\sum_{i=1}^{n}x_i^4\right)-\left(\sum_{i=1}^{n}x_i^3\right)^2}. \qquad (17)$$

In expressions (16)-(17) - $u_i, y_i\ (i=1,2,\ldots,n)$ are the values of the system's input and output variables in the steady state at the moment $t_i\ (i=1,2,\ldots,n)$.

*B. Identification of dynamic parameter*

The application of the Fourier approximation [20] for the output periodic signal of the Hammerstein model enables to obtain the estimates of the Fourier coefficients $\hat{a}_0/2$, $\hat{a}_k$, $\hat{b}_k$, $(k=1,2)$.

By equating the estimate $\hat{a}_0/2$ with its theoretical values from (14), we'll get:

$$\frac{\hat{a}_0}{2}=\frac{c_1(1-c_1)^2B+c_2B^2}{(1-c_1)^3}+\frac{c_2A^2(1+\omega^2T^2)}{2(1-c_1)\left[(1-c_1)^2+\omega^2T^2\right]}. \tag{18}$$

From (18) we get:

$$\hat{a}_0(1-c_1)^3\left[(1-c_1)^2+\omega^2T^2\right]=\left[2c_1(1-c_1)^2B+2c_2B^2\right]\times$$
$$\times\left[(1-c_1)^2+\omega^2T^2\right]+c_2(1-c_1)^2(1+\omega^2T^2)A^2. \tag{19}$$

After the transformation of expression (19) we get:

$$\left[\hat{a}_0(1-c_1)^3+(c_2A^2+2c_1B)(1-c_1)^2-2c_2B^2\right]\omega^2T^2=$$
$$=(1-c_1)^2\left[c_2A^2+2c_1(1-c_1)^2B-\hat{a}_0(1-c_1)^3\right]. \tag{20}$$

Using the expression (20) at different frequencies $\omega=\omega_i\ (i=1,2,\ldots,n)$, we obtain:

$$\left[\hat{a}_{0i}(1-c_1)^3+(c_2A^2+2c_1B)(1-c_1)^2-2c_2B^2\right]\omega_i^2T_0+$$
$$+\varepsilon_i=(1-c_1)^2\left[c_2A^2+2c_1(1-c_1)^2B-\hat{a}_{0i}(1-c_1)^3\right], \tag{21}$$

where $\hat{a}_{0i}\ (i=1,2,\ldots,n)$ - values of the Fourier coefficient at the frequency $\omega_i$ and $\varepsilon_i\ (i=1,2,\ldots,n)$ - errors of measurements and approximations, and

$$T_0=T^2. \tag{22}$$

Let's consider the features for $T$ parameter estimation by the method of least squares using the expression (21).

The error squared sum is:

$$S=\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}\left\{(1-c_1)^2\left[c_2A^2+2c_1(1-c_1)^2B-\hat{a}_{0i}(1-c_1)^3\right]-\right.$$
$$\left.-\left[\hat{a}_{0i}(1-c_1)^3-(c_2A^2+2c_1B)(1-c_1)^2+2c_2B^2\right]\omega_i^2T_0\right\}^2. \tag{23}$$

If we differentiate (23) by $T_0$, we'll obtain:

$$\frac{dS}{dT_0}=2\sum_{i=1}^{n}\left\{(1-c_1)^2\left[c_2A^2+2c_1(1-c_1)^2B-\hat{a}_{0i}(1-c_1)^3\right]-\right.$$
$$\left.-\left[\hat{a}_{0i}(1-c_1)^3-(c_2A^2+2c_1B)(1-c_1)^2+2c_2B^2\right]\omega_i^2T_0\right\}\times$$
$$\times\left[\hat{a}_{0i}(1-c_1)^3-(c_2A^2+2c_1B)(1-c_1)^2+2c_2B^2\right]\omega_i^2. \tag{24}$$

Equating (24) to zero, we'll obtain the following expression for estimating $\hat{T}_0$:

$$\sum_{i=1}^{n}\left[\hat{a}_{0i}(1-\hat{c}_1)^3-(\hat{c}_2A^2+2\hat{c}_1B)(1-\hat{c}_1)^2+2\hat{c}_2B^2\right]^2\omega_i^4T_0=$$
$$=\sum_{i=1}^{n}\left\{(1-\hat{c}_1)^2\left[\hat{c}_2A^2+2\hat{c}_1(1-\hat{c}_1)^2B-\hat{a}_{0i}(1-\hat{c}_1)^3\right]\right\}\times$$
$$\times\left[\hat{a}_{0i}(1-\hat{c}_1)^3-(\hat{c}_2A^2+2\hat{c}_1B)(1-\hat{c}_1)^2+2\hat{c}_2B^2\right]\omega_i^2. \tag{25}$$

If we take into account (22), from the expression (25) we get:

$$\hat{T}=\sqrt{\frac{\sum_{i=1}^{n}M_iN_i\omega_i^2}{\sum_{i=1}^{n}N_i^2\omega_i^4}}, \tag{26}$$

where

$$M_i=(1-\hat{c}_1)^2\left[\hat{c}_2A^2+2\hat{c}_1(1-\hat{c}_1)^2B-\hat{a}_{0i}(1-\hat{c}_1)^3\right],$$

$$N_i=\left[\hat{a}_{0i}(1-\hat{c}_1)^3-(\hat{c}_2A^2+2\hat{c}_1B)(1-\hat{c}_1)^2+2\hat{c}_2B^2\right].$$

$\hat{T}$ for Hammerstein models can be also obtained by using expressions $\hat{a}_k$, $\hat{b}_k$, $(k=1,2)$ and compare the estimates obtained using $\hat{a}_0$ .

## VI. ACCURACY OF THE RECEIVED RESULTS

To use the algorithms of parameter identification, designed in accordance with the developed identification methods in the manufacturing conditions under noise and disturbances, it is necessary to investigate the identification method on accuracy.

The identification method was investigated by theoretical analysis and computer modeling. The reliability of the received results, at the identification of the closed-loop nonlinear systems of industrial processes conditions in the presence of noise and errors, depends on the measurement accuracy of the system's input and output signals and on the mathematical processing of the experimental data. When using various schemes of the numerical harmonic analysis, it is recommended to accept that the value of the output signal at a certain time moment is an estimation of the mathematical expectation of the value of the output function at the present time moment.

In figure 4. is given the scheme for the Hammerstein model with unit feedback.

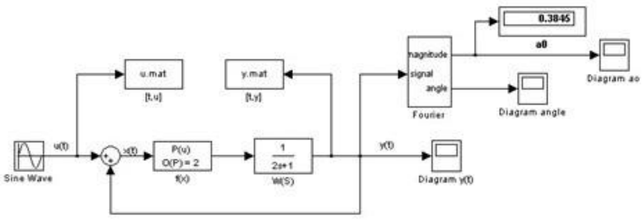


Fig. 4. Scheme for Hammerstein model with unit feedback

Besides, as it is known, that used method of the least squares for parameter estimation is noiseless.

The investigation of the algorithms of parameter identification of nonlinear systems was carried out by means of computer modeling using MATLAB.

We used both the tool package Simulink-toolbox for the system modeling and the tool Symbolic Math Toolbox for the solution of the equations.

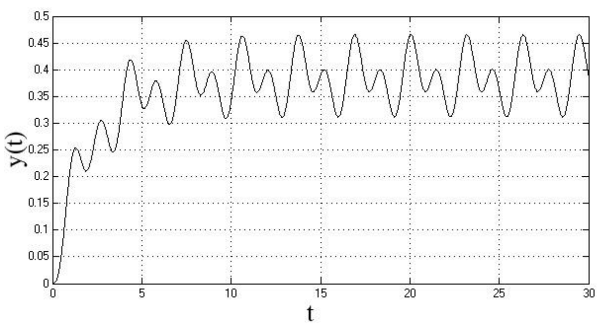


Fig. 5. Diagram of the output signal of the Hammerstein model with unit feedback

During the experiment, at the parameter values: T=2, $c_1$=0.1, $c_2$ = 0.03, the following parameter values were obtained for this model by calculations: T=1.9996, c= 0.0997, c2 =0.0204.

Computer modeling confirmed the correctness of the results obtained from the theoretical studies and the expediency of using them to solve practical problems.

## VII. CONCLUSIONS

The identification problems of nonlinear systems get specific forms, proceeding from their functioning peculiarities. When carrying out the experiment in such systems it is necessary to consider the conditions of stability of each movement at the output, since such movements can become steady under certain values of the system's static parameters. Besides, when obtaining the expressions of the output variables of the closed nonlinear systems analytically it is necessary to solve the nonlinear differential equation that relates to the mathematical difficulties.

The method of parameter identification of nonlinear systems with feedback in the steady state at their representation by the Hammerstein model with unit feedback on the basis of active experiments under input sinusoidal influences is offered in the given work. Analytical expressions of forced oscillations obtained at the system output are used to solve parameter identification problems.

At the parameter identification under the determined input influences static parameters are estimated first, followed by dynamic parameters estimation based on the least squares method. The developed parameter identification method is investigated on accuracy.

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