Small Satellite Attitude Estimation with Uncertain Process Noise

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***Abstract*—In this study the Unscented Kalman Filter (UKF) and Singular Value Decomposition (SVD) methods are integrated in the nontraditional attitude filtering algorithm to estimate a small satellite's attitude. Influence of the process noise bias type system changes to the innovation of UKF is investigated. It is proved that the bias type process noise change may be converted to the mean square of innovation of UKF and such type of changes can be compensated using the covariance scaling techniques.**

Keywords—attitude estimation, process noise, unscented Kalman filter, nanosatellite, magnetometer, Sun sensor

I. Introduction

Sun sensor and magnetometer onboard small spacecraft flying low Earth altitudes are typically used as attitude sensors. Employing a Kalman filtering approach, the measurements from magnetometers and Sun sensors are fused within a satellite dynamics propagation model. This integrated process allows for estimating the satellite's attitude, serving as a fundamental solution to the problem of attitude estimation. The nonlinear measurements of reference directions can be used to create a Kalman filter for satellite attitude and rate estimates in traditional approaches [1,2]. The nonlinear models of the reference directions serve as the foundation for the measurement models in the filter. Therefore, nonlinear equations are used to link the data and states.

When employing a non-traditional method, the attitude angles are initially computed using vector measurements and a method that determines orientations in a single frame at each recursive step. Subsequently, these attitude angles are directly utilized as measurement inputs for an attitude filter, such as the Extended Kalman filter (EKF) [3-5] or Unscented Kalman filter (UKF) [5-7].

The Unscented Kalman Filter and Singular Value Decomposition (SVD) methods are integrated in the nontraditional attitude filtering algorithm to estimate a nanosatellite's attitude. The SVD approach determines the attitude of the nanosatellite and provides one estimate at a single frame utilizing measurements from the magnetometer and Sun sensor as the initial stage of the algorithm. These attitude terms are subsequently fed into the UKF with their error covariances.

It is shown in this study that the process noise bias type system changes will cause a change in the statistical characteristics of the innovation of UKF. The theoretical basics of the Q-adaptive SVD-aided UKF with uncertain process noise mean are developed and presented. For the purpose of estimating a nanosatellite's attitude, simulations are compared using the adaptive and non-adaptive versions of the nontraditional attitude filter in the presence of process noise bias.

II. Satellite Rotational Motion and Attitude Measurement Models

By using the quaternion attitude representation, the satellite's kinematics equation of motion can be expressed as,

$$\dot{\boldsymbol{q}}(t)=\frac{1}{2}\Omega(\boldsymbol{\omega}_{BR}(t))\boldsymbol{q}(t) \quad (1)$$

Here $\boldsymbol{q}$ consists of four attitude parameters in the quaternion, $\boldsymbol{q}=\begin{bmatrix}q_1 & q_2 & q_3 & q_4\end{bmatrix}^T$. Since the last term is a scalar and the first three terms are vector terms, we may rewrite the quaternion as $\boldsymbol{q}=\begin{bmatrix}\boldsymbol{g}^T & q_4\end{bmatrix}^T$, $\boldsymbol{g}=\begin{bmatrix}q_1 & q_2 & q_3\end{bmatrix}^T$, $\Omega(\boldsymbol{\omega}_{BR})$ is the skew symmetric matrix having the components of $\boldsymbol{\omega}_{BR}$ angular velocity vector of body frame with respect to the reference (orbital) frame.

It is important to specify the body's angular rate vector in relation to the inertial axis frame independently from the angular velocity vector, $\boldsymbol{\omega}_{BI}=\begin{bmatrix}\omega_x & \omega_y & \omega_z\end{bmatrix}^T$. $\boldsymbol{\omega}_{BI}$ and $\boldsymbol{\omega}_{BR}$ can be related via,

$$\boldsymbol{\omega}_{BR}=\boldsymbol{\omega}_{BI}-A\begin{bmatrix}0 & -\omega_o & 0\end{bmatrix}^T. \quad (2)$$

Here ω_o indicates the satellite's angular orbital velocity, A is the attitude matrix. In (3) the attitude matrix A and the quaternions are related by the relation,

$$A=(q_4^2-|\boldsymbol{g}|^2)I_{3\times3}+2\boldsymbol{g}\boldsymbol{g}^T-2q_4\left[\boldsymbol{g}\times\right], \quad (3)$$

where I_{3x3} is the identity matrix with the dimension of 3×3 and $\left[\boldsymbol{g}\times\right]$ is the skew-symmetric matrix whose elements are the components of $\boldsymbol{g}$ vector.

Based on Euler's equations, it is possible to deduce the satellite's dynamic equations,

DOI: https://doi.org/10.54381/pci2023.05

$$J\frac{\omega_{BI}}{dt} = N_d - \omega_{BI} \times (J\omega_{BI}), \quad (4)$$

where J is the principal moments of inertia matrix as $J = diag(J_x, J_y, J_z)$ and N_d is the vector of disturbance torque affecting the nanosatellite.

This study outlines our approach for attitude determination on a nanosatellite that incorporates both magnetometers and a Sun sensor as attitude sensors. These sensors are considered classic examples since they provide unit vector measurements. To determine the attitude, it is necessary to establish the unit vectors in the reference orbit frame that correspond to the unit vectors measured by the sensors in the spacecraft body frame. The measurement models presented in this study depict the relationships between these computed and measured unit vectors, facilitating accurate attitude determination.

The magnetometer measuring model can be described as follows,

$$\boldsymbol{B}_b = A\boldsymbol{B}_o + \eta_1. \quad (5)$$

Here $\boldsymbol{B}_b$ is the measured magnetic field vector in the body frame, $\boldsymbol{B}_o$ is the calculated magnetic field vector in the orbit frame and η_1 is the assumed to be zero-mean Gaussian white noise.

The Sun direction measurement model in orbital frame can be expressed as follows

$$\boldsymbol{S}_b = A\boldsymbol{S}_o + \eta_2. \quad (6)$$

Here $\boldsymbol{S}_b$ is the measured Sun direction vector in the body frame, $\boldsymbol{S}_o$ is the calculated Sun direction vector in the orbit frame and η_2, which is taken to be zero-mean Gaussian white noise. It is assumed that that the magnetometers and Sun sensors are calibrated against any bias and/or misalignment.

III. Integration of Singular Value Decomposition and Unscented Kalman Filter for Attitude Estimation

The nontraditional attitude estimation procedure that is composed of two stages as singular value decomposition (SVD) and unscented Kalman filter (UKF) is presented in this section.

A. Singular Value Decomposition Method

Single-frame attitude estimate techniques include the Singular Value Decomposition (SVD, q-method, QUEST, FOAM, and others. Due to its greater robustness, the SVD approach is chosen as the single-frame method in this instance [8].

Given a set of $n \geq 2$ vector measurements, $\hat{u}_B^i$, in the body system, choosing to minimize the loss function given as for an ideal attitude matrix, *A*, is one possibility,

$$J(A) = \sum_{i=1}^{n} w_i \left|\hat{u}_B^i - A\hat{u}_R^i\right|^2, \quad (7)$$

where w_i is the weight of the i[th] vector measurement, $\hat{u}_R^i$ is the vector in the reference coordinate system.

When we express the loss function as (7), the problem reduces to the problem of maximizing the trace, $\mathrm{tr}(AB^T)$. Among the single-frame attitude estimate algorithms, the SVD is one of the most accurate, dependable, and robust techniques, as thoroughly addressed in [8].

Decomposition of the matrix B into singular values,

$$B = U\Sigma^T V^T = U\mathrm{diag}\left[\Sigma_{11} \quad \Sigma_{22} \quad \Sigma_{33}\right]V^T, \quad (8)$$

where *U* and *V* are orthogonal and the singular values obey $\Sigma_{11} \geq \Sigma_{22} \geq \Sigma_{33} \geq 0$. The trace is therefore maximized for,

$$U^T A_{opt} V = \mathrm{diag}[1 \quad 1 \quad \det(U)\det(V)], \quad (9)$$

and the optimal rotation matrix is,

$$A_{opt} = U\mathrm{diag}[1 \quad 1 \quad \det(U)\det(V)]V^T. \quad (10)$$

By looking at the covariance matrix for rotation angle error, the accuracy of the estimated A_{opt} can be known. If we first define $s_1 = \Sigma_{11}$, $s_2 = \Sigma_{22}$, $s_3 = det(U)det(V)\Sigma_{33}$ then the covariance matrix P_{svd} is calculated as,

$$P_{svd} = U\mathrm{diag}[(s_2+s_3)^{-1} \quad (s_3+s_1)^{-1} \quad (s_1+s_2)^{-1}]U^T \quad (11)$$

B. Unscented Kalman Filter for Rotational Motion Estimation

The Unscented Transform, which is a deterministic sampling technique, is utilized in our approach to obtain a reduced set of sample points (or sigma points) from the prior mean and covariance of the states. This technique forms the basis of the Unscented Kalman Filter (UKF). These sigma points are transformed nonlinearly. The altered sigma points are used to calculate the posterior mean and covariance [9].

Since discrete-time nonlinear equations are used to construct the UKF, the following equations represent the process model,

$$x(k+1) = f(x(k), k) + w(k), \quad (12)$$

$$y(k) = Hx(k) + v(k). \quad (13)$$

Here, $x(k)$ is the state vector and $y(k)$ is the measurement vector. Moreover $w(k)$ and $v(k)$ are the process and measurement error noises, which, according to the assumption, are processes with Gaussian white noise with zero mean and a covariance of $Q(k)$ and $R(k)$ respectively, H is the measurement matrix of system. The unscented Kalman filter is presented in Appendix A.

IV. Influence of Process Noise Bias to the Innovation

In this Section influence of process noise bias type system changes to the innovation of UKF is investigated.

TABLE I. RMS ERROR FOR SAUKF AND ASAUKF FOR PROCESS NOISE BIAS CASE

RMS Error	**Process Noise Bias**					
	δ_{low}		δ_{medium}		δ_{high}	
	SaUKF	***ASaUKF***	***SaUKF***	***ASaUKF***	***SaUKF***	***ASaUKF***
q_1	0.00186	0.00270	0.00198	0.00273	0.00223	0.00284
q_2	0.00317	0.00253	0.00501	0.00429	0.00752	0.00675
q_3	0.00192	0.00189	0.00247	0.00227	0.00323	0.00283
q_4	0.00585	0.00509	0.00743	0.00660	0.00956	0.00869

Assumption. The process noise is assumed to be biased and its average value is given by $E\left[w(k)\right] = \delta(k)$.

It is clear that in this case the system state estimates will be biased.

Theorem: If the measurements are processed by the UKF a process noise bias occurs at the iteration step $k = \tau$, then at the all $k > \tau$ steps the estimation and innovation of UKF are biased and innovation bias is equal to the observed prediction bias.

A sampling covariance matrix of innovation is presented as statistics for detecting and compensating for changes in process noise. The sample covariance matrix of the innovation $\nu(j)$ may be expressed as follows

$$\hat{S}_{\nu}(k) = \frac{1}{M} \sum_{j=k-M+1}^{k} \nu(j)\nu^T(j) \tag{14}$$

where M is the width of the "sliding window".

If there exists a bias in the mean of the innovation at time τ, we denote the biased innovation as $\nu_b(k)$, then the biased innovation is defined as,

$$\nu_b(k) = \nu(k)\,,\ \ k = 1,2,\ldots \tau - 1 \tag{15}$$

$$\nu_b(k) = \nu(k) + \mu(k)\,,\ \ k = \tau, \tau + 1, .. \tag{16}$$

where μ(.) is an unknown bias vector. In the case of $k \geq \tau$, in the sample innovation covariance (14) a biased innovation $\nu_b(k) = \nu(k) + \mu(k)$ is used instead of an unbiased innovation $\nu(k)$, where $\mu(k)$ is the innovation bias.

Remark. Note that the mean of the innovation $\nu_b(k)$ in this case is not zero, therefore the formula (14) with $\nu_b(k)$ is not a sample covariance. In the "sliding window" it is the mean square of innovation (MSI).

Statement. The biased innovation $\nu_b(k)$ leads to an increase in the mathematical expectation of the mean square of innovation.

It can be seen from the *Theorem* and the *Statement* above that the process noise bias is transferred to the innovation bias and changes the mean square of innovation. As a result, the bias in the process noise is transferred to the MSI. Thus, the MSI can be chosen as a monitoring statistic.

V. Q-Adaptive Integrated SVD/UKF Attitude Estimation Algorithm

The SVD approach runs as the initial stage of the algorithm and provides one estimate per single frame for the nanosatellite attitude. The UKF is then provided with these estimated attitude terms as input. As a result, the satellite's attitude is calculated (see Fig.1). 'SVD-aided UKF' is called 'SaUKF' for short. 'Q-Adaptive SVD-aided UKF' is called 'ASaUKF' throughout the text for brevity.

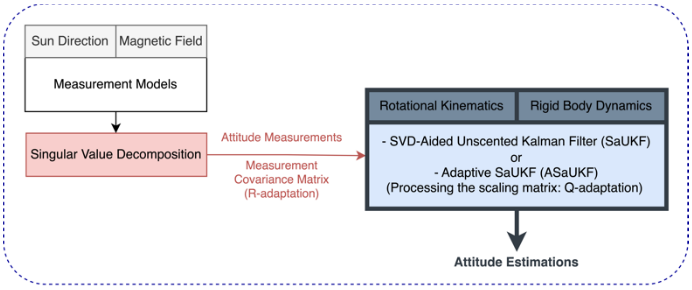

Fig. 1. Flow chart of SaUKF and ASaUKF algorithms.

The estimation covariance from the SVD is also input into the UKF method and used as the filter's measurement noise covariance matrix in addition to the estimated quaternions, i.e. $R(k+1) = P_{svd}(k+1)$. Because of this, the nontraditional filter is intrinsically robust to measurement noise increments in particular. The SVD's estimation covariance, which is also the filter's measurement noise covariance, increases in the presence of a measurement error, allowing the filter to function without being significantly affected [7].

The adjustment of the filter's process noise covariance matrix, Q presents one challenge for nontraditional attitude filters. When the environment changes, it's very important to optimize the process noise covariance. These changes include adjustments to the inertia parameters (as when the satellite enters or exits an eclipse) and adjustments to the disturbance torques. In this study, the scaling matrix $\Lambda(k)$ is defined as

$$\begin{aligned}
&\Lambda(k) = \left[H^T(k+1)H(k+1)\right]^{-1} H^T(k+1) \times \\
&\left[\frac{1}{\mu}\sum_{j=k-\mu+1}^{k} \nu(j+1)\nu^T(j+1) - H(k+1)P^*(k+1/k)H^T(k+1) - R(k+1)\right] \\
&\times H(k+1)\left[Q(k)H^T(k+1)H(k+1)\right]^{-1}
\end{aligned} \tag{17}$$

Here, $P^*(k+1|k)$ is the predicted covariance without the additive process noise.

An adaptive technique is used to tweak the UKF in order to adapt it to the changing environment. The adaptation rule is simple to implement because the measurement model is linear.

VI. Analysis of Simulation Results

A tumbling small satellite is considered for the analysis in order to test the presented algorithms under crucial measurement and environmental challenges. The nanosatellite has the principal moment of inertia $J = \mathrm{diag}\begin{bmatrix}0.055 & 0.055 & 0.017\end{bmatrix}$ kg m^2. The algorithm runs for almost 1 orbital period with 1 Hz sampling rate for the filter and the sensors. The sensors are selected as three-axis magnetometers and three-axis Sun sensors corrupted by the standard deviations of $\sigma_B = 300$ nT for magnetometers and $\sigma_S = 0.002$ for Sun sensors. To demonstrate how the filter behaves that is integrated with a single-frame method in these intervals, an eclipse period is introduced between 500th and 1500th seconds. SaUKF and ASaUKF are compared for the adaptation of process noise bias between 4500th and 5500th seconds.

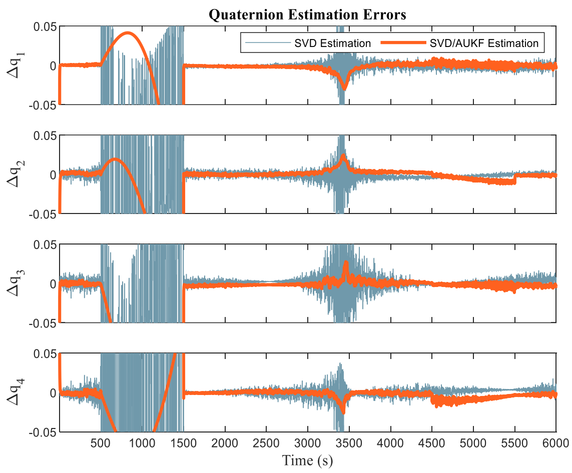

Fig. 2. Quaternion estimation errors of ASaUKF under process noise bias applied as δ_{high}.

For investigating the filter's ability to adapt against process bias faults the algorithm is tested for three different faults: low, medium and high noise bias for a short period of time. Process noise biases are identified by adding the constant bias term to the process noise between 4500th and 5500th s. For the test case the constant term is selected as $\delta = \{\delta_{\text{low}} \quad \delta_{\text{medium}} \quad \delta_{\text{high}}\}$ to represent low, medium, and high bias cases:

$$\delta_{\text{low}} = \begin{bmatrix}0.005 & 0.005 & 0.005 & 0 & 0 & 0\end{bmatrix}^T;$$
$$\delta_{\text{medium}} = \begin{bmatrix}0.008 & 0.008 & 0.008 & 0 & 0 & 0\end{bmatrix}^T;$$
$$\delta_{\text{high}} = \begin{bmatrix}0.012 & 0.012 & 0.012 & 0 & 0 & 0\end{bmatrix}^T.$$

The Root Mean Squares Errors (RMS) for SaUKF and ASaUKF algorithms for process noise bias cases are shown in Table 1. The estimation errors from low to high levels of process noise bias show increase of error. In Fig. 2, quaternion estimations of ASaUKF and SaUKF algorithms are given when having noise biases of δ_{high} level. As seen, using adaptive algorithm, the attitude quaternions can be estimated reasonably well. The obtained results show that the bias type process noise changes can be compensated using the covarince scaling techniques.

VII. Conclusions

In this study, an attitude filtering technique is proposed that adapts the process noise uncertainties. The bias and noise increment type process noise uncertainties are taken into consideration. To begin with, the Unscented Kalman Filter (UKF) and Singular Value Decomposition (SVD) methods are combined to estimate a nanosatellite's attitude. Influence of the process noise bias type system changes to the innovation of UKF is investigated. It is proved that the bias type process noise change may be converted to the mean square of innovation of UKF and such type of changes can be compensatd using the covarince scaling techniques.

Various levels of process noise bias type system changes are tested. Simulations are compared using the adaptive and non-adaptive versions of the nontraditional attitude filter. The simulation results show that, in the cases of process noise bias, the multiple fading factors based adaptive SVD-aided UKF can adapt to the changing environment better than the SaUKF.

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Appendix A

The unscented Kalman filter is well known and can be found elsewhere (e.g. [9]). In this appendix we restate the fundamental UKF equations applied to the attitude and rate estimation problem of nanosatellite.

The UKF algorithm's first phase is deciding the $2n+1$ sigma points with a mean of $\hat{x}(k|k)$ and a covariance of $P(k|k)$. For an n dimensional state vector, these sigma points are obtained by,

$$x_0(k|k) = \hat{x}(k|k), \quad \text{(A.1a)}$$

$$x_\gamma(k|k) = \hat{x}(k|k) + \left(\sqrt{(n+\kappa)P(k|k)}\right)_\gamma, \quad \text{(A.1b)}$$

$$x_{\gamma+n}(k|k) = \hat{x}(k|k) - \left(\sqrt{(n+\kappa)P(k|k)}\right)_\gamma, \quad \text{(A.1c)}$$

where, $x_0(k|k)$, $x_\gamma(k|k)$ and $x_{\gamma+n}(k|k)$ are sigma points, n is the state number, and κ is the scaling parameter which is employed for fine tuning. $\left(\sqrt{(n+\kappa)P(k|k)}\right)_\gamma$ corresponds to the γ^{th} column of the indicated matrix and γ is given as $\gamma = 1 \ldots n$.

The UKF procedure's subsequent step is to assess the transformed set of sigma points for each of the points using,

$$x_l(k+1|k) = f\left[x_l(k|k), k\right].\ l = 0 \ldots 2n \quad \text{(A.2)}$$

The anticipated mean and covariance are then calculated using these transformed values [9].

$$\hat{x}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa x_0(k+1|k) + \frac{1}{2}\sum_{l=1}^{2n} x_l(k+1|k)\right\}$$

,(A.3a)

$$\begin{aligned} P(k+1|k) &= P^*(k+1|k) + Q(k) = \\ &\frac{1}{n+\kappa}\left\{\kappa\left[x_0(k+1|k) - \hat{x}(k+1|k)\right]\left[x_0(k+1|k) - \hat{x}(k+1|k)\right]^T\right. \\ &\left.+\frac{1}{2}\sum_{l=1}^{2n}\left[x_l(k+1|k) - \hat{x}(k+1|k)\right]\left[x_l(k+1|k) - \hat{x}(k+1|k)\right]^T\right\} \\ &+Q(k) \end{aligned}$$

(A.3b)

Here, $\hat{x}(k+1|k)$ is the predicted mean, $P(k+1|k)$ is the predicted covariance, $P^*(k+1|k)$ is the predicted covariance without the additive process noise.

Furthermore, the predicted observation vector is,

$$\hat{y}(k+1/k) = H\hat{x}(k+1/k)\ . \quad \text{(A.4)}$$

The observation covariance matrix is then calculated as follows,

$$P_{yy}(k+1/k) = HP(k+1/k)H^T\ . \quad \text{(A.5)}$$

The cross-correlation matrix, on the other hand, can be found as,

$$P_{xy}(k+1/k) = P(k+1/k)H^T. \quad \text{(A.6)}$$

The UKF algorithm's update step comes next. The residual term $\nu(k+1)$ (or innovation sequence) is established at that stage as the difference between the actual observation and the predicted observation, initially by employing measurements $y(k+1)$,

$$\nu(k+1) = y(k+1) - \hat{y}(k+1|k), \quad \text{(A.7)}$$

The innovation covariance is,

$$\begin{aligned} P_{\nu\nu}(k+1|k) &= P_{yy}(k+1|k) + R(k+1) = \\ &HP(k+1/k)H^T + R(k+1) \end{aligned} \quad \text{(A.8)}$$

Here $R(k+1)$ is the measurement noise covariance matrix. Equation is used to calculate the Kalman gain,

$$K(k+1) = P_{xy}(k+1|k)P_{\nu\nu}^{-1}(k+1|k). \quad \text{(A.9)}$$

Finally, the covariance matrix and updated states are determined by,

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)\nu(k+1), \quad \text{(A.10)}$$

$$\begin{aligned} P(k+1|k+1) &= P(k+1|k) - \\ &P_{xy}(k+1/k)P_{\nu\nu}^{-1}(k+1|k)P_{xy}{}^T(k+1/k). \end{aligned} \quad \text{(A.11a)}$$

or

$$\begin{aligned} P(k+1|k+1) &= P(k+1|k) - \\ &K(k+1)P_{\nu\nu}(k+1|k)K^T(k+1). \end{aligned} \quad \text{(A.11b)}$$

Here, $\hat{x}(k+1|k+1)$ is the estimated state vector and $P(k+1|k+1)$ is the estimated covariance matrix.