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Actuarial Risk Modeling Using Bayesian Approach

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***Abstract*— The problem of correct analysis and mathematical description of financial risks is considered; some uncertainties relevant to the risks analysis are highlighted, and an example is given for the financial process model construction in the form of prognostic distribution. The data used characterize the payment flow within a five year period for a chosen insurance company**

Keywords*— *financial risks; actuarial models; mathematical modeling; Bayesian approach; actual data; forecasting distribution

I. Introduction

One of the most urgent problems that are to be solved in financial risk management is development and practical application of adequate mathematical models. The models should be clear for practitioners and extended or modified when necessary with new structural elements, correct prior expert estimates of parameters and initial conditions, extra experimental evidences, possible areas of application etc. A substantial role in correct and timely solving of the modeling and management of financial risks plays so called system approach that is in taking into consideration of current market factors, detecting of new influential factors (including the latent ones), determining the levels and frequencies of influence of internal and external factors on the process under study. The approach also suggests determining and taking into consideration the uncertainties of structural, statistical and parametric nature, correct problem statement and solving the optimization problems where it is necessary, and application of analytical quality criteria at each stage of data analysis [1 – 3].

II. Financial Risk Analysis

The purpose of the study is as follows: – to determine the types of actuarial risks and to select the forms of their possible mathematical description; – to consider the possibilities for application of Bayesian approach to actuarial risks model constructing; to construct a Bayesian type model for a stochastic financial process. The actuarial field is characterized with a wide set of possible risks, to be more exact as follows: (1) individual risks; (2) collective risks for one (short) period; (3) collective risks for a long period; (4) large distributed risks of losses; (5) operational risks; (6) the solvency risk; (7) risks of ruin, and others. It is well known that high losses may take place today due to widely spread operational risks. The losses can be determined as direct and indirect ones that appear due to inappropriate organization of work (operations) or inadequate flow of internal processes inside an insurance company, incorrect behavior of personal and/or inappropriate functioning of technical support systems, or influence of external factors. The situational analysis linked directly to the sources of financial risks should be grounded on objective information regarding current state of a company from reliable independent sources. It may turn out that a specific company does not have enough information for detecting and management all possible risks. In such cases the company should perform an extra analysis directed towards the following issues: (1) determining exact moment of time when the financial losses took place as well as identification of the fact of decision making that resulted in the losses; (2) estimation of possible revenue that company could get in a case of avoiding the risky situation; (3) distribution of actual financial losses among to several risk factors if such exists; (4) collection of extra information from the personal that is directly involved into the risky situation that took place.

Generally the procedure of risk type identification and management could be represented in the form of the following cyclical sequence of actions: (1) determining all possible types risks for a company; (2) identification, thorough understanding and description of all the situations that are favorable for the risk factor generation; (3) a profound analysis of possible risk types with determining possible level of losses and methods for their estimation and forecasting; (4) managerial decision making directed towards management of specific risks types; (5) continuous monitoring of managerial decision implementation, detection and analysis of indicators of possible risks; (6) writing a detailed report regarding the performed actions directed towards elimination, ignoring or active control of situations with possible risks.

III. Constructing a Model Example for Random Payments

Let $\{x(k)\}$ is a random process of payments coming, where k is a discrete moment of payment. Denote the payment accumulation for the first moment of time as $\exp(x(1))$, and for an arbitrary time moment k the fund accumulation will be: $\exp(x(1)+x(2)+...+x(k))$. For the convenience of further derivations denote the argument under exponent as follows: $y(k)=\sum_{i=1}^{k} x(i)$. The problem is to determine distribution types for $y(k)$ and $F(k) = \exp(y(k))$. One of the simplest models to describe behavior of such financial process is the first order autoregression AR(1): $x(k) = a_0 + a_1 x(k-1) + \varepsilon(k)$, where $\varepsilon(k)$ is the process of normal random disturbances. Let's represent the equation in the form:

$$x(k) - \mu = a(x(k-1) - \mu) + \sigma\, z(k), \qquad (1)$$

where μ is a mean value for respective time series data; a, σ are model parameters; $\{z(k)\} \sim Norm\,(0,1)$ is a sequence of independent equally distributed random values. Note that equation (1) is a discrete analog for the first order ordinary differential equation. It is necessary to find expressions for parameter estimates using maximum likelihood approach. The conditional likelihood function for the payment data series can be written as [1]:

$$f(x|\mu,\sigma^2,a) = \prod_{i=-N+2}^{0} \left\{ \begin{array}{l} (2\pi\sigma^2)^{-1/2} \times \\ \times \exp\left[-\dfrac{1}{2\sigma^2}\left(x(i) - \mu - a(x(i-1) - \mu)\right)^2 \right] \end{array} \right\}.$$

And the final expression for the process $y(k)$ is as follows:

$$\begin{array}{l} y(k)\,|\,\mu,\sigma^2,a,x = \mu k + (x(0) - \\ -\mu)\,M(a,k) + \sigma(V(a,k))^{1/2} Z \end{array},$$

where

$$M(a,k) = a(1 - a^k)/(1 - a);$$

$$V(a,k) = \frac{1}{(1-a)^2} * \\ * \left(k - \frac{2a(1-a^k)}{1-a} + \frac{a^2(1-a^{2k})}{1-a^2} \right)$$

IV. Computing Results

To perform necessary computing experiments the data were selected that characterize payments accumulation during a five year period for a chosen insurance company. As a result of the computations performed we got the following parameter estimates that characterize the data:

$$\hat{a} = -\,0{,}2587;\;\; \hat{\mu} = 0{,}2165;\;\; \hat{\sigma}^2 = 0{,}0873.$$

The estimates computed by the method of moments for the same data are as follows: $\hat{\mu} = 0{,}2139$; $\hat{\sigma}^2 = 0{,}0861$. Thus the values computed by both methods are close to each other what gives a ground for the statement that the model constructed has satisfactory degree of adequacy to the process under study. There are possibilities for the further improvement of the model by making use of more complicated structures that could be derived from deeper statistical analysis of the data available.

V. Conclusions

Thus, the actuarial field is characterized with a wide set of possible risks such as: individual risks and collective risks for short period; collective risks for a long period; large distributed risks of losses; operational and solvency risks; risks of ruin and others. All the risks require a special approach to their analysis, modeling, forecasting and management. In spite of availability of wide set of mathematical and statistical models there is a need in refinement of existing and development of new types of models that would cover the field of financial risks. An advantage of the proposed method is in a possibility of determining the forecasting posterior distributions (and appropriate forecasts) for the process under study. The further study is directed towards the use of more complicated model structures to improve the model adequacy and quality of forecasts resulting from it.

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