Optimization of Interval Length for Neural Network Based Fuzzy Time Series

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***Abstract* — Fuzzy time series models have become important in past decades with neural networks. Hence, this study aims to improve forecasting performance of neural network based fuzzy time series by using an optimization function to interval length which affects forecasting accuracy. So, a new approach for improving forecasting performance of neural network-based fuzzy time series is applied with optimization process. The empirical results show that the model with proposed approach by optimization of interval length outperforms other forecasting models proposed in the literature.**

Keywords* — *Fuzzy time series; neural networks; optimization; forecasting; interval length

I. Introduction

Fuzzy time series definition was introduced by Song and Chissom [1-2]. After Song and Chissom, many fuzzy time series models have been used to improve solves for different kind of problems such as enrollment [3-6], temperature forecasting [7], stock index forecasting [4], [7].

In this study, we have used a new approach based on Yu and Huarng's method (2010) [8] algorithm with some changes and modifications for improving forecasting performance of neural network-based fuzzy time series. Also, differing from previous studies, this study decides the length of intervals by using an effective approach which optimizes the length of intervals by using MATLAB instead of using constant length of intervals as 100 or 1000 by following previous studies for given data set forecasting.

II. Methods

Conventional time series refer to real numbers, but fuzzy time series are structured by fuzzy sets [3]. Song and Chissom first proposed the definitions of fuzzy time series [1-2]. A fuzzy set is a class of objects with a continuum of grade of membership. The some general definitions of fuzzy time series are given as follows:

1. Let U be the universe of discourse, where

$$U=\{u_1,u_2,...,u_n\} \tag{1}$$

and u_i are possible linguistic values of U, then a fuzzy set of linguistic variables A_i of U is defined by

$$A_i=\mu_{A_i}(u_1)/u_1+\mu_{A_i}(u_2)/u_2+\cdots+\mu_{A_i}(u_n)/u_n \tag{2}$$

where μ_{A_i} is the membership function of the fuzzy set A_i,

$$\mu_{A_i}:U\rightarrow[0,1]. \tag{3}$$

u_k is an element of fuzzy set A_i and $\mu_{A_i}(u_k)$ is the degree of belongingness of u_k to A_i.

$$\mu_{A_i}(u_k)\in[0,1] \quad \text{and} \quad 1\le k\le n. \tag{4}$$

2. Let $Y(t)$ which is a subset of real numbers be the universe of discourse defined by the fuzzy set $f_i(t)$. If $F(t)$ consists of

$$f_i(t) \quad (i=1,2,...), \tag{5}$$

then $F(t)$ is defined as a fuzzy time series on $Y(t)$.

3. If there exists a fuzzy relationship

$$R(t-1,t), \tag{6}$$

such that

$$F(t)=F(t-1)\times R(t-1,t), \tag{7}$$

where $\times$ is an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by

$$F(t-1)\rightarrow F(t). \tag{8}$$

4. Suppose

$$F(t-1)=A_i \tag{9}$$

and

$$F(t)=A_j, \tag{10}$$

a fuzzy logical relationship is defined as

$$A_i\rightarrow A_j, \tag{11}$$

where A_i is named as left-hand side of the fuzzy logical relationship and A_j the right-hand side.

In this study, we used a new approach which based on Yu and Huarng's algorithm (2010) [8] but has some changes, modifications and is summarized as follows:

1. Difference and Adjustment: This method forecasts the differences between every two consecutive observations instead of using observations directly.

2. Universe of Discourse: The universe of discourse can be defined with two proper positive numbers. Suppose the length of interval is set to l. Then U is separated into equal length of intervals. Define the linguistic values of the fuzzy sets.
In this step, we used MATLAB function for determining the length of intervals called "fminbnd".

3. Fuzzification: Next, d'(t-1,t) can be fuzzified into a set of degrees of membership, V(t-1,t),

$$V(t-1,t)=\left[\mu_{t-1,t}^{1},\mu_{t-1,t}^{2},\ldots\right].$$

Here μ is the membership function. Next, we suggested an algorithm for this step to make fuzzification easily and practically.

4. Neural Network Training and Forecasting: To improve forecasting results, this method uses all the degrees of membership to establish fuzzy relationships.
This method then applies back-propagation neural networks to establish (or train) the fuzzy relationships. It has one input layer, one or more hidden layers and one output layer.

5. Defuzzification: Weighted averages method is applied to defuzzify the degrees of membership.

6. Forecasting: Once the forecasted difference between t-1 and t is obtained, the forecast for t can be calculated. For performance evaluation, the mean squared error (MSE) is calculated.
For performance evaluation, the mean squared error (MSE) is calculated as follows:

$$MSE=\frac{\sum_{t=k+1}^{n}\left(forecast(t)-obs(t)\right)^{2}}{n-k}, \qquad (14)$$

where there are n observations, including k in-sample and n-k out-of-sample observations.
Determining the length of intervals is important issue in fuzzy time series forecasting. It affects the performance and improves forecasting accuracy. So, in this study, we used a MATLAB function called "fminbnd" which minimizes MSE for optimization process as [9]. The algorithm used by "fminbnd" is based on golden section search and parabolic interpolation.

III. Conclusion

In this study, we used various degrees of membership with neural networks. So, more information was considered in training and forecasting processes. A neural network based fuzzy time series forecasting approach is applied with based on the optimization of the interval length for analyzing models. In fuzzification step, an algorithm of finding all V(t-1,t) is proposed to improve forecasting performance of neural network-based fuzzy time series models. Various numbers of hidden nodes, which are important for forecasting while using neural networks, are taken into account.

MSE values of other proposed methods available in the literature such as Song and Chissom (1993) [1], Song and Chissom (1994) [10], Sullivan and Woodall (1994) [11], Chen(1996) [3], Hwang et al. (1998) [12], Chen (2002) [13], Huarng (2001) [4], Aladag et al. (2009) [14], Yu and Huarng (2010) [8], Tsaur et al. (2005) [15], Li and Chen (2004) [16] and Li and Cheng (2007) [17] are given for comparison purpose in Table I.

Empirical results show that the neural network-based fuzzy time series model with proposed method and optimized the length of interval outperformed the other best and most commonly used fuzzy time series forecasting models proposed in the literature.

TABLE I. MSE Values Of Other Proposed Methods

Method	Order	MSE
Song and Chissom(1993)	1	412499
Song and Chissom(1994)	1	775687
Sullivan and Woodall (1994)	1	386055
Chen(1996)	1	407507
Hwang et al. (1998)	5	278919
Huarng (2001)	1	78792
Chen (2002)	3	86694
Aladag et al. (2009)	2	78073
Yu and Huarng (2010)	1	247461
Tsaur et al. (2005)	1	134923
Li and Chen (2004)	1	173459
Li and Cheng (2007)	1	85040

References

[1] Q. Song, B. S. Chissom (1993), Fuzzy Time Series and its Models, Fuzzy Sets and Systems, Vol.54, pp.269-277.

[2] Q. Song, B. S. Chissom (1993), Forecasting Enrollments with Fuzzy Time Series, Fuzzy Sets and Systems, Part 1, Vol.54, pp.1-9.

[3] S.-M. Chen (1996), Forecasting Enrollments Based on Fuzzy Time Series, Fuzzy Sets and Systems, Vol.81, No. 3, pp.311-319.

[4] K. Huarng (2001), Heuristic Models of Fuzzy Time Series for Forecasting, Fuzzy Sets and Systems, Vol.123, No.3, pp.369-386.

[5] K. Huarng, H. K. Yu (2006), Ratio-Based Lengths of Intervals to Improve Fuzzy Time Series Forecasting, IEEE Trans. Syst., Man, Cybern. B, Cybern., Vol. 36, No. 2, 328-340.

[6] Q. Song, R. P. Leland, B. S. Chissom (1995), A New Fuzzy Time-Series Model of Fuzzy Number Observations, Fuzzy Sets and Systems, Vol. 73, pp. 341-348.

[7] L.-W. Lee, L.-H. Wang, S.-M. Chen (2007), Temperature Prediction and TAIFEX Forecasting Based on Fuzzy Logical Relationships and Genetic Algorithms, Expert Systems with Applications, Vol. 33, pp. 539-550.

[8] T. H.-K. Yu, K.-H. Huarng (2010), A Neural Network-Based Fuzzy Time Series Model to Improve Forecasting, Expert Systems with Applications, 37, 3366-3372.

[9] E. Egrioglu, C. H. Aladag, U. Yolcu, V. R. Uslu, M. A. Basaran (2010), Finding an optimal interval length in high order fuzzy time series, Expert Systems with Applications, 37, 5052-5055.

[10] Q. Song and B. S. Chissom (1994), Forecasting Enrollments with Fuzzy Time Series, Part 2, Fuzzy Sets and Systems, 62, pp. 1-8.

[11] J. Sullivan and W. H. Woodall (1994), A Comparison of Fuzzy Forecasting and Markov Modeling, Fuzzy Sets and Systems, 64(3), pp. 279-293.

[12] J. R. Hwang, S.-M. Chen, C.-H. Lee (1998), Handling Forecasting Problems Using Fuzzy Time Series, Fuzzy Sets and Systems, Vol. 100, No. 1-3, pp. 217-228.

[13] S.-M. Chen (2002), Forecasting Enrollments Based on High-Order Fuzzy Time Series, Cybernetics and Systems, 33(1), pp. 1-16.

[14] C. H. Aladag, M. A. Basaran, E. Egrioglu, U. Yolcu, V. R. Uslu (2009), Forecasting in High Order Fuzzy Time Series by Using Neural Networks to Define Fuzzy Relations, Expert Systems with Applications, 36, pp. 4228-4231.

[15] R.-C. Tsaur, J.-C. Yang, H.-F. Wang (2005), Fuzzy relation analysis in fuzzy time series model, Computers & Mathematics with Applications, 49, 539–548.

[16] S.-T. Li and Y.-P. Chen (2004), Natural partitioning-based forecasting model for fuzzy time-series, Proceedings IEEE International Conference on Fuzzy Systems, 3, 1355-1359.

[17] S.-T. Li and Y.-C. Cheng (2007), Deterministic fuzzy time series model for forecasting enrollments, Computers & Mathematics with Applications, 53, 1904-1920.