Problem of Determination of Leakages in Oil Pipeline Networks

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Abstract— **The problem of determination of breakpoints and of volume of oil leaks under non-stationary regimes of transportation over a linear part of a main pipeline is investigated in the work. The problem of identification is considered in the frame of class of optimal control problems and the formulas for the gradient of the target functional are obtained for the considered problem. These formulas allow using the numerical methods of the first order to solve the problem. The results of numerical experiments are given.**

Keywords— transportation of hydrocarbon raw material; optimal control problem; leak of fluid; volume of oil leaks

I. INTRODUCTION

At transportation of hydrocarbon raw material the basic pipelines are used. Development of a network of the main oil pipelines is accompanied by wide introduction of modern means of computer technologies, systems of telemechanics on process of swapping.

Leak of hydrocarbon raw material from pipelines represents serious danger to people and environment. All this turns around greater economic losses and leads to ecological accidents. Determination of places and volume of leak at operation of the pipeline always represented a difficult problem. In most cases for the control of a condition of the pipeline special value gets the research directed on creation of ways and devices of detection of leak from pipelines [1-2]. But in the pipeline located in a ground when survey is impossible, it is possible to determine the presence of leak on the presence of change of pressure in the pipeline. However such method not always gives trustworthy information because of influence of non-uniform distribution of temperature along the pipeline on size of measured pressure in it. Now growth of interest to new decisions on monitoring and localizations of places of leak is observed. The classification of most popular methods of finding leaks from pipelines is given in [4], their advantages and disadvantages are also shown in the work.

We investigate a problem of the localization of the fluid leakage under non-stationary regimes of transportation as a problem of parametrical identification of the system with distribution parameters in the work. The initial data for identification problem is continuous observational regimes at different points, as well as at the ends of the part of pipeline, where the existence of the leak of oil is possible.

II. STATEMENT OF THE PROBLEM

We investigate a problem of the determination of breakpoints and of volume of oil leaks under non-stationary regimes of transportation over a linear part of a main pipeline. For subsonic speeds when it is possible to neglect change of high-speed pressures and to accept density $\rho = const$ without a practical error. So we receive the linearized system for functions of mass speed and pressure.

We rewrite the linearized system of differential equations of unsteady laminar flow of fluid with constant density over the linear part of pipeline with constant diameter and length l in the following form as [3]:

$$\begin{cases} -\dfrac{\partial P(x,t)}{\partial x} = \dfrac{\partial Q(x,t)}{\partial t} + 2aQ(x,t), \\ -\dfrac{\partial P(x,t)}{\partial t} = c^2 \dfrac{\partial Q(x,t)}{\partial x}. \end{cases} \quad (1)$$

where $P(x,t)$, $Q(x,t)$ – pressure and fluid flow rate at the point $x \in (0,l)$ of the pipeline at the moment time t, c – sound speed in the environment, it is possible to consider that the kinematical coefficient of the viscosity - γ without a practical error does not depend on pressure and the condition $2a = \dfrac{32\gamma}{d^2} = const$ appears, thus, quite exact for a laminar stream of a drop fluid, d – diameter.

We assume that there is permanent long-term observation over the expenditure and pressure at the ends of the pipeline, (flow-meters of turbine or volumetric type, connected distance with the computer which is being on the central control office) i.e. the following conditions are given

$$Q(0,t) = Q_{in}(t), Q(l,t) = Q_{out}(t), \quad (2)$$

$$P(0,t) = p_{in}(t), P(l,t) = p_{out}(t), \quad (3)$$

at $t > t_0$, t – gets enough large value. It is important to note, that if process (1) or (3) operates long enough as a result of friction inherent in any real physical system, the influence of initial data eventually weaken. Therefore over a long period of observation over the tracking process, i.e. at the interval $[t_0, T]$ the influence of initial values of regimes decreases for a current state of the process at $t = t_0$ and there is such

τ that, for $t > \tau$ at greater essential there is an influence of values of regional conditions, for any time interval $[t_0, T]$ where the value of τ is defined by property of course of process, value of regional conditions and factor and we'll assume that $\tau < T$. Thus, we naturally come to a problem without entry conditions [5, c.106-111].

Let's assume that at some moments of time $t_i \geq t_0$ at any m points $\xi_i \in (0,l)$, $i = 1,...,m$ of the pipeline has started leak of a fluid with charge $q_i^{loss}(t), i = 1,.,m$. We'll assume that $\min_{1\le i\le m} t_i = t_0$ no underrating of generality. Thus the flow of oil over the considered part is described by $m+1$ systems of differential equations of the form (1), separately for each part or interval: $x \in (\xi_i, \xi_{i+1})$, $i = 0,...,m$, $\xi_0 = 0, \xi_{m+1} = l$. In addition to conditions (2) and (3) on the ends of each part the next conditions of conjugation are fulfilled:

$$P(\xi_i - 0, t) = P(\xi_i + 0, t),$$

$$Q(\xi_i - 0, t) = Q(\xi_i + 0, t) + q_i^{loss}(t), \; i = 1,...,m .$$

We can write the description of flow of a fluid using the generalized Dirac delta function by a system of the following form [5]:

$$\begin{cases} -\dfrac{\partial P(x,t)}{\partial x} = \dfrac{\partial Q(x,t)}{\partial t} + 2aQ(x,t), \\ -\dfrac{\partial P(x,t)}{\partial t} = c^2 \dfrac{\partial Q(x,t)}{\partial x} + c^2 \sum_{i=1}^{m} q_i^{loss}(t)\delta(x-\xi_i). \end{cases} \quad (4)$$

Clearly from (1) that one of conditions (2) or (3) is enough to calculate the modes $P(x,t;\xi,q(t)), Q(x,t;\xi,q(t))$ of movement in the pipeline, at known places and charges of leak.

In the case of unknown places and charges of leak $(\xi_i, q_i^{loss}(t))$, $i = 1,...,m$ we'll assume that at some given points $\tilde{x}_j$, $j = 1,...,L$, $0 = \tilde{x}_0 < ... < \tilde{x}_j < \tilde{x}_{j+1} < .... < \tilde{x}_L = l$ is permanent long-term observation over the pressure, besides the ends of part of the pipeline, i.e. the values of

$$P^i{}_{mes}(t), \; i = 1,..., L, \; P_{in}(t) = p^0_{mes}(t), P_{out}(t) = p^L_{mes}(t)$$

are also given. It is naturally to assume that the searching places of leaks aren't coincide with points of observation over the modes. We assume that between two points of oservation there can be only one place of leakage, i.e. for any i, $i = 1,...,m$ one can find such $s_i \in \{1,...,L\}$, that: $\tilde{x}_{s_i} < \xi_i < \tilde{x}_{s_i+1}$. This conditions is necessary for uniquness of determination of places and charges of fluid leak. If there are two places of leak in any interval $(\tilde{x}_j, \tilde{x}_{j+1})$, then the places of unknown leaks in this interval is determined no uniquly, irrespective other conditions of observation.

The problem consists of finding of the places of fluid leaks $\xi = (\xi_1,...,\xi_m)$, $\xi_i \in [0,l]$, $i = 1,...,m$, and corresponding values of fluid leakage $q^{loss}(t) = (q_1^{loss}(t),...,q_m^{loss}(t))$ at $t \in [t_0, T]$ using considered mathematical model and observational data.

With purpose to solve the problem let's consider the following functional determining value of difference between supervised and calculated régimes:

$$J(\xi, q^{loss}(t)) = \sum_{i=0}^{L} \int_{\tau}^{T} [P(\tilde{x}_i, t) - P^i{}_{mes}(t)]^2 dt + \\ + \varepsilon_1 \left\| q^{loss}(t) - \tilde{q}^0 \right\|^2_{L_2^m[T-\Delta,T]} + \varepsilon_2 \left\| \xi - \tilde{\xi} \right\|^2_{R^m} \to \min, \quad (5)$$

where $P(x,t) = P(x,t;\xi,q^{loss})$ is the solution to the problem of (2)-(4) for any given $(\xi, q^{loss}(t))$; $[\tau, T]$ is the interval of observation over the process, which regimes are not depend on entry conditions at $t \geq \tau$; $\tilde{\xi}, \tilde{q}^0 \in R^m, \varepsilon_1, \varepsilon_2$ are the regularization parameters. At that the initial conditions do not function any more till the moment t_0 in the interval $[\tau, T]$. That is why the precise value of the initial value t_0 is not a matter of principle.

We'll assume that that proceeding from technological and technical reasons there are restrictions on identified functions and parameters, for example,

$$0 < \xi_i \leq \xi_{i+1} \leq l, \; \underline{q} \leq q_i^{loss}(t) \leq \overline{q}, \; i = 1,...,m, \; t \in [t_0, T], \quad (6)$$

where $\underline{q}_j, \overline{q}_j$ are given.

The considered problem (3)-(6) on determination places and charges of leaks $(\xi_i, q_i^{loss}(t))$, $i = 1,...,m$ is the problem of parametrical optimal control of object described by the hyperbolic system. We use numerical methods based on iteration procedures of optimization of the first order (gradient or conjugate gradient projection) to solve the problem, for example

$$\begin{pmatrix} \xi \\ q^{loss}(t) \end{pmatrix}^{k+1} = \Pr_{(6)}\left[\begin{pmatrix} \xi \\ q^{loss}(t) \end{pmatrix}^{k} - \alpha \, grad\, J(\xi^k, (q^{loss}(t))^k) \right], \; k = 0,1,..., \quad (7)$$

where $\Pr_{(6)}(\cdot)$ is a projection operator of vector on the set defined by the constraints (6); α is a step of one-dimensional search in the line of the anti-gradient of the target functional [6]. We need to obtain the formulas for the gradient of the target functional to fulfill the procedure (7), what the next part will initiate.

Let's notice that, we'll solve the problem (3)-(6) and if at the end we'll get that, $\left| q_i^{loss}(t) \right| \leq \varepsilon$, $t \in [\tau, T]$, here ε is enough little number, then it means that, there isn't a leakage at s_i-th part $[\tilde{x}_{s_i}, \tilde{x}_{s_i+1}]$ of pipeline.

III. The Formulas for the Gradient of the Target Functional

We can use the method of variation of the optimizable parameters to obtain the formulas for the gradient of the functional [6].

Let $\psi(x,t) = (\psi_1(x,t), \psi_2(x,t))$ be the solution to the following adjoint boundary problem

$$-\frac{\partial \psi_1(x,t)}{\partial x} = \frac{\partial \psi_2(x,t)}{\partial t} - 2\sum_{i=1}^{L-1} [P(x,t) - P^i_{mes}(t)]\delta(x - \tilde{x}_i) \quad (8)$$

$$-\frac{\partial\psi_1(x,t)}{\partial t}=c^2\frac{\partial\psi_2(x,t)}{\partial x}-2a\psi_1(x,t), \quad (9)$$

$$\psi_1(x,T)=0,\ \psi_2(x,T)=0,\ x\in(0,l), \quad (10)$$

$$\psi_1(0,t)=2[P(0,t;u)-P^0_{mes}(t)],\ t\in[\tau,T], \quad (11)$$

$$\psi_1(l,t)=-2[P(l,t;u)-P^L_{mes}(t)],\ t\in[\tau,T]. \quad (12)$$

The formulas for the components of the gradient of the target functional with respect to the control functions $q^{loss}(t)$ are determined in the following form:

$$grad_{q_i^{loss}(t)}J(\xi,q^{loss}(t))=c^2\int_\tau^T\psi_2(\xi_i,t)dt+2\varepsilon_1\int_\tau^T(q_i^{loss}(t)-\overline{q}_i^0)dt,$$

$$i=1,\dots,m. \quad (13)$$

$$grad_{\xi_i}J(\xi,q^{loss}(t))=c^2\int_\tau^T q_i^{loss}(t)(\psi_2(x,t))'_x|_{x=\xi_i}dt+2\varepsilon_2(\xi_i-\overline{\xi}_i),$$

$$i=1,\dots,m. \quad (14)$$

IV. NUMERICAL EXERIMENTS

Let's consider the next especially formulated test problem for which the results of numerical experiments are given below.

We'll assume that the process (the work regime of pumping stations at the ends of pipeline's part) of transportation of oil was observing during 12 minutes. The fluid's kinematic viscosity is $\nu=1{,}5\cdot10^{-4}\,(m^2/\sec)$, density is $\rho=920\,(kg/m^3)$ on the part of the pipeline with diameter $d=530\ (mm)$ and length $l=100\ (km)$. $2a=\frac{32\nu}{d^2}=0{,}017$ in this case is true.

Let the fluid flow rate at the ends of part of the pipeline is determined by the next functions:

$$Q(0,t)=150\sin t+400\ \ (m^3/hour),$$

$$Q(l,t)=150\sin t+200\ \ (m^3/hour).$$

We'll assume that the place of leak is on the point $\xi=30\ (km)$, but the charge of the leak is determined by the function $q^{loss}(t)=450\sin 0{,}15t\ \ (m^3/hour)$ to formulate test observational values of fluid flow rate at the ends of part. The boundary problem (3), (4) was numerically solved using these data and the numerical values of fluid flow rate functions were determined at the ends of part: $P(0,t)$, $P(l,t)$. Then these values were used to formulate the target functional (6), but the place and charge of leak ξ, $q^{loss}(t)$ was "forgotten".

It is required to define ξ, $q^{loss}(t)$ by using the method of solving the problem (3)-(6) suggested above in the work. For this purpose the conjugate gradient projection method was used. The numerical solution of the boundary problem (3)-(4) was fulfilled by applying the implicit scheme of net method with steps $h_x=0{,}01$, $h_t=0{,}1$ [7].

The obtained results of minimization of functional (6) for different initial values of identifiable parameters $(\xi,\ q^{loss}(t))^0$, in addition to the required number of iterations of the conjugate gradient method (the number of one-dimensional minimization) are given on the table 1.

TABLE 1 THE NUMERICAL RESULTS TO SOLUTION OF TEST PROBLEM

N	q^0 $m^3/hour$	ξ^0 km	ξ^* km	J^0	J^*	**The number of iter.**
1	**162**	**70**	**31,6**	**239,7**	**0,0088**	**36**
2	**144**	**20**	**28,5**	**198,2**	**0,0098**	**28**
3	**288**	**40**	**30,1**	**1843,8**	**0,0009**	**94**
4	**148**	**70**	**30,0**	**204,8**	**0,0009**	**68**
5	**360**	**70**	**30,2**	**3526,2**	**0,0009**	**152**
6	**36**	**20**	**29,9**	**360,9**	**0,0009**	**42**
7	**108**	**90**	**31,8**	**123,9**	**0,0097**	**54**

The accurate and obtained plots of values of the leakage functions are given on the figure 1.

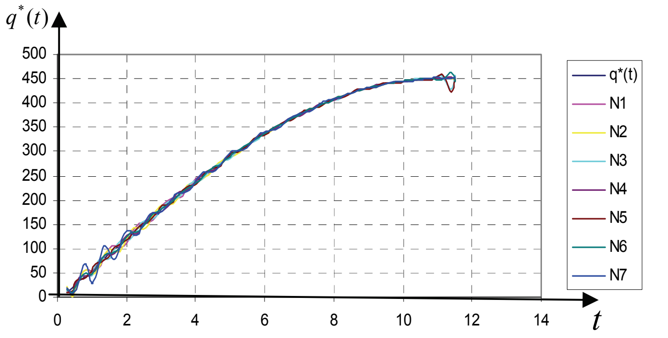

Fig.1 The plots of obtained leak functions $q^{loss}(t)$ for seven different entry values of optimal control $\xi_0, q_0^{loss}(t)$.

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