Investigation of Quasi-singular Controls in Optimal Control Problem Described by Volterra Type Integro-Differential Equations

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***Abstract*— An optimal control problem described by a system of Volterra type integro-differential equations is considered. Necessary condition of optimality of quasi-singular controls is proved.**

Key words*— *Volterra type equations; control problem; optimal control; necessary conditions; quasi-singular control

I. Introduction

In the papers [1-5] and etc. difference necessary optimality conditions of first order were obtained for an optimal control problem described by the systems of Volterra type integro- differential equations.

In the present paper, necessary optimality conditions of quasi-singular controls are obtained.

II. Statement of Problem

Assume that it is required to minimize the multipoint functional

$$S(u)=\varphi(x(T_1),x(T_2),\ldots,x(T_k)), \tag{1}$$

under constraints

$$u(t)\in U\subset R^r,\quad t\in T=[t_0,t_1], \tag{2}$$

$$\dot{x}(t)=f(t,x(t),u(t))+\int_{t_0}^{t}K(t,\tau,x(\tau),u(\tau))d\tau,\; t\in T,$$

$$x(t_0)=x_0. \tag{3}$$

Here $f(t,x,u)$, $K(t,\tau,x,u)$ are the given n-dimensional vector-functions continuous in totality of variables together with partial derivatives with respect to (x,u) to second order inclusively, t_0,t_1,x_0, $T_i\in(t_0,t_1)$, $(t_0<T_1<T_2<\ldots<T_k\le t_1)$, $i=\overline{1,k}$ are given, U is a given nonempty, bounded and convex set, $u(t)$ is an r-dimensional piecewise (with finite number of discontinuity points) vector of control actions (admissible control), $\varphi(a_1,a_2,\ldots,a_k)$ is a given twice differentiable scalar function.

III. Necessary Optimality Conditions

Assuming $(u(t),x(t))$ a fixed admissible process, introduce the denotation

$$H(t,x(t),u(t),\psi(t))=\psi'(t)f(t,x(t),u(t))+$$
$$+\int_{t}^{t_1}\psi'(\tau)K(\tau,t,x(t),u(t))d\tau,$$

where $\psi=\psi(t)$ is an n-dimensional vector-function of conjugated variables being a solution of the equation

$$\psi(t)=\int_{t}^{t_1}H_x(\tau,x(\tau),u(\tau),\psi(\tau))d\tau-$$
$$-\sum_{i=1}^{k}\alpha_i(t)\frac{\partial\ \ (x(T_1),x(T_2),\ldots,x(T_k))}{\partial a_i}.$$

Here $\alpha_i(t)$ is a characteristic function of the segment $[t_0,T_i]$.

Theorem 1. (linearized maximum principle). For optimality of the admissible control $u(t)$ it is necessary that the inequality

$$H_u'(\theta,x(\theta),u(\theta),\psi(\theta))(v-u(\theta))\le 0,$$

be fulfilled for all $\theta\in[t_0,t_1)$ and $v\in U$.

The linearized maximum condition for different problems of optimal control of integro-differential equations was proved in the papers [1- 5] and others.

Definition 1. The admissible control $u(t)$ is said to be a classic control if for all $\theta\in[t_0,t_1)$ and $v\in U$

$$H_u'(\theta,x(\theta),u(\theta),\psi(\theta))(v-u(\theta))=0.$$

Assume that by definition

$$Q(t,\tau)=F(t,\tau)f_u(\tau,x(\tau),u(\tau))+$$
$$+\int_\tau^t F(t,s)K_u(s,\tau,x(\tau),u(\tau))ds,$$

$$M(\tau,s)=$$
$$=-\sum_{i,j=1}^k \alpha_i(\tau)\alpha_j(s)Q'(T_i,\tau)\frac{\partial^2\ (x(T_1),\ldots,x(T_k))}{\partial a_i\partial a_j}\times$$
$$\times Q(T_j,s)+\int_{\max(\tau,s)}^{t_1} Q'(t,\tau)H_{xx}(t,x(t)u(t),\psi(t))Q(t,s)ds.$$

Here $F(t,\tau)$ is the analogy of the Cauchy matrix for a linearized problem being a solution of the problem

$$F_\tau(t,\tau)=-F(t,\tau)f_x(\tau,x(\tau),u(\tau))-$$
$$-\int_\tau^t F(t,s)K_x(s,\tau,x(\tau),u(\tau))d\tau,$$

$$F(t,t)=E, \quad (E \text{ is a } (n\times n) \text{ unique matrix}).$$

The following theorem is proved by means of the method suggested in the papers [6, 7]

Theorem 2. (integral necessary condition of optimality of quasi-singular controls). For optimality of the quasi-singular control $u(t)$ in problem (1)-(3), it is necessary that the inequality

$$\int_{t_0}^{t_1}\int_{t_0}^{t_1}(v(\tau)-u(\tau))'M(\tau,s)(v(s)-u(s))dsd\tau+$$
$$+\int_{t_0}^{t_1}(v(t)-u(t))H_{uu}(t,x(t),u(t),\psi(t))\times$$
$$\times(v(t)-u(t))dt+$$
$$+2\int_{t_0}^{t_1}\left[\int_{t_0}^{t_1}(v(\tau)-u(\tau))'H_{ux}(\tau,x(\tau),u(\tau),\psi(\tau))Q(\tau,t)d\tau\right]\times$$
$$\times(v(t)-u(t))dt\le 0 \tag{4}$$

be fulfilled for all $v(t)\in U,\ t\in T$.

IV. Conclusion

Necessary optimality conditions of quasi-singular controls are obtained for an optimal control problem described by a system of Volterra type integro-differential equations.

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