Investigation of Output Flows in the System with Parallel Service of Multiple Requests

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***Abstract*— In this paper we have built a model with parallel service of requests. System consists of two blocks of service with infinite number of servers, and input Poisson flow with parameter λ of double requests (first and second types). We have found analytical formula for generating function of two-dimensional output flow. It allows to obtain numerical characteristics of output flow.**

***Keywords* — parallel service, Poisson flow of double requests, two-dimensional output flow.**

I. Introduction

Queueing systems with infinite number of servers can be used as a models of real systems in different field of daily life: banking, insurance, transport etc.

Most of papers on investigation of the systems with infinite number of servers devoted to the study of number of busy servers. However, practically it is necessary to know characteristics of such flows, study the properties of output flows isn't sufficiently developed [1].

II. Mathematical Model

In this paper we consider system with two blocks of service (fig. 1), each of which contains infinite number of servers, and with input Poisson flow with parameter λ of double requests [2]. It means that two requests arrive in the system at the same time in the moment of event occurrence in our flow.

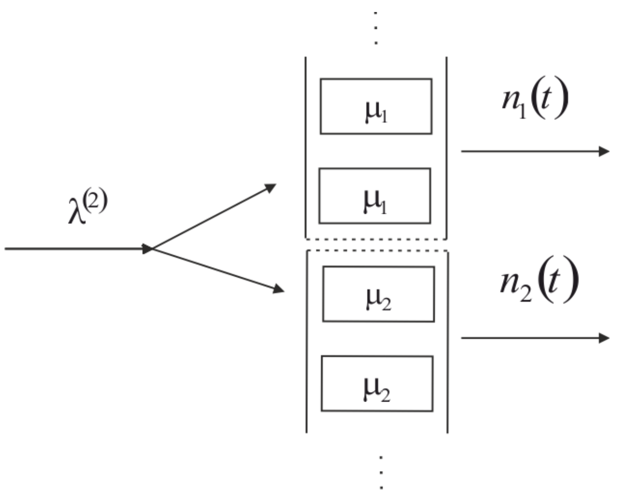

Figure 1. *Queueing system with parallel service of double requests*

Service discipline assumes that one of these requests comes in the first block, another request comes in the second block and occupies any of the empty servers for service. Service times for both requests are exponentially distributed with parameters μ_1 and μ_2.

The task is investigation of two-dimensional random process $\{n_1(t),n_2(t)\}$ of the number of served requests from each block.

III. Investigation of Output Flow

Two-dimensional random process $\{n_1(t),n_2(t)\}$ is a non-Markovian process. Therefore, we introduce additional components $i_1(t),i_2(t)$, its define number of requests in the first and second blocks.

Denote by $P\{i_1(t),i_2(t),n_1(t),n_2(t)\}=P(i_1,i_2,n_1,n_2,t)$ the probability distribution of its vales.

One can easily show that the probability distribution for considering process is a solution of following Kolmogorov's differential equations [3]:

$$\partial P(i_1,i_2,n_1,n_2,t)/\partial t=-(\lambda+i_1\mu_1+i_2\mu_2)P(i_1,i_2,n_1,n_2,t)+$$
$$+(i_1+1)\mu_1P(i_1+1,i_2,n_1-1,n_2,t)+(i_2+1)\mu_2P(i_1,i_2+1,n_1,n_2-1,t)+$$
$$+\lambda P(i_1-1,i_2-1,n_1,n_2,t)$$

Here we have initial conditions for solution of this system

$$P\{i_1,i_2,n_1,n_2\}=R(i_1,i_2),$$

where $R(i_1,i_2)$ is initial two-dimensional distribution of the number of busy servers in the block of service. It equals to final distribution.

Let denote generating function [4] of the process $\{i_1(t), i_2(t), n_1(t), n_2(t)\}$ as follows:

$$F(x_1,x_2,y_1,y_2,t)=\sum_{i_1}\sum_{i_2}\sum_{n_1}\sum_{n_2}x_1^{i_1}x_2^{i_2}y_1^{n_1}y_2^{n_2}P(i_1,i_2,n_1,n_2,t),$$

we obtain differential equation:

$$\frac{\partial F(x_1,x_2,y_1,y_2,t)}{\partial t}+\frac{\partial F(x_1,x_2,y_1,y_2,t)}{\partial x_1}(x_1-y_1)\mu_1+$$
$$+\frac{\partial F(x_1,x_2,y_1,y_2,t)}{\partial x_2}(x_2-y_2)\mu_2=\lambda(x_1x_2-1)F(x_1,x_2,y_1,y_2,t).$$

The initial conditions have a form:

$$F\{x_1,x_2,y_1,y_2\}=f(x_1,x_2).$$

Obtain the function $f(x_1,x_2)$ by the method of generating function. Thus, we have:

$$f(x_1,x_2)=\exp\left\{\frac{\lambda(x_1-1)(x_2-1)}{\mu_1+\mu_2}+\frac{\lambda(x_1-1)}{\mu_1}+\frac{\lambda(x_2-1)}{\mu_2}\right\}.$$

Solving following differential equation

$$\frac{dt}{1}=\frac{dx_1}{\mu_1(x_1-y_1)}=\frac{dx_2}{\mu_2(x_2-y_2)}=\frac{dF(x_1,x_2,y_1,y_2,t)}{\lambda(x_1x_2-1)F(x_1,x_2,y_1,y_2,t)}.$$

We find first and second integrals from the equations:

$$dt=dx_1/\mu_1(x_1-y_1),\qquad dt=dx_2/\mu_2(x_2-y_2)$$

It is clear that

$$x_1=y_1+C_1e^{\mu_1 t},\ C_1=(x_1-y_1)e^{-\mu_1 t}$$
$$x_2=y_2+C_2e^{\mu_2 t},\ C_2=(x_2-y_2)e^{-\mu_2 t}.$$

Let obtain the last integral from the equation

$$dt=\frac{dF(x_1,x_2,y_1,y_2,t)}{\lambda(x_1x_2-1)F(x_1,x_2,y_1,y_2,t)},$$

then, it takes the form

$$F(x_1,x_2,y_1,y_2,t)=\Phi(C_1,C_2)\exp\left\{\lambda C_1C_2\frac{e^{(\mu_1+\mu_2)t}}{\mu_1+\mu_2}+\lambda C_1(y_2-1)\frac{e^{\mu_1 t}}{\mu_1}+\right.$$
$$\left.+\lambda C_2(y_1-1)\frac{e^{\mu_2 t}}{\mu_2}+\lambda(y_1y_2-1)t+\lambda(y_1-1)t+\lambda(y_2-1)t+\lambda C_2\frac{e^{\mu_2 t}}{\mu_2}+\lambda C_1\frac{e^{\mu_1 t}}{\mu_1}\right\},$$

where $\Phi(C_1,C_2)$ is stationary probability distribution of the number of busy servers in each block, which is obtained by using following formula

$$\Phi(C_1,C_2)=\exp\left\{-\lambda C_1C_2\frac{1}{\mu_1+\mu_2}-\lambda C_1(y_2-1)\frac{1}{\mu_1}-\lambda C_2(y_1-1)\frac{1}{\mu_2}-\lambda C_2\frac{1}{\mu_2}-\right.$$
$$\left.-\lambda C_1\frac{1}{\mu_1}+\lambda(C_1+y_1-1)(C_2+y_2-1)\frac{1}{\mu_1+\mu_2}+\lambda(C_1+y_1-1)\frac{1}{\mu_1}\lambda(C_2+y_2-1)\frac{1}{\mu_2}\right\}.$$

Assuming that $x_1=1$ and $x_2=1$, we obtain formula for generating function of two-dimensional output flow

$$F(y_1,y_2,t)=\exp\left\{\frac{-\lambda\mu_1(1-e^{-\mu_1 t})(y_1-1)(y_2-1)}{(\mu_1+\mu_2)\mu_2}-\frac{\lambda\mu_2(1-e^{-\mu_2 t})(y_1-1)(y_2-1)}{(\mu_1+\mu_2)\mu_1}+\right.$$
$$\left.+\lambda(y_1y_2-1)t+\lambda((y_1-1)+(y_2-1))t\right\}.$$

This analytical equation allows us to find ordinary stationary characteristics of output flow:

Mean value:

$$M_{y1}=My2=\lambda t.$$

Dispersion:

$$D_{y1}=D_{y2}=(\lambda t)^2.$$

Correlation coefficient:

$$r(y_1,y_2)=\frac{\mu_1(1-e^{-\mu_1 t})}{(\mu_1+\mu_2)\mu_2}+\frac{\mu_2(1-e^{-\mu_2 t})}{(\mu_1+\mu_2)\mu_1}.$$

In this paper, we have obtained analytical formula for joint generating function of number of servers in blocks and served requests. It allows us to find exact numerical characteristics of random vector. From the form of generating function we have done conclusion that output flows are dependent, and investigation of its flows should be done only in conjunction. We have found formulae for mean value, dispersion and correlation coefficient of the number of attempts to one of the blocks of service under arbitrary initial conditions. Results allow to solve any practical problems of application of mathematical model.

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