Probabilistic – Timing Performances of a Wireless Local Network with a Controlled Access Protocol

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***Abstract* — In this work a wireless local network of IEEE 802.11 standard and its families 802.11 b, 802.11 d and 802.11 n is considered at using controlled access protocol in them and the methods for calculation of their probabilistic-timing performances are suggested.**

Keywords— wireless local networks; Laplace operator; controlled access protocol; station

For the last years wireless local networks (WLN) of information transmission gain growing popularity, this is associated with ease and rapidity of their scanning, service simplicity and other advantages [1]. Besides, among wireless local networks the most propagation is related to wireless local networks of IEEE 802.11 standard and its other families: 802.11b, 802.11d, 802.11n etc.

In these networks two typical operating conditions are considered: structured operating conditions and "ad hoc" (point-to-point) operating conditions.

Wireless local networks with structured operating conditions are composed of two links [1]: wire local network and wireless stations connected to it by an access point. Therefore such networks are not "pure" wireless networks.

Wireless networks with "ad hoc" operating conditions don't have supporting wire infrastructure. In these operating conditions stations directly interact with each other, without using a special access point. While using these operating conditions the provision of network infrastructure is not required [1]. Therefore a wireless local network with "ad hoc" operating conditions is selected for analysis.

These networks correspond to communication systems incorporating herewith radio channel and subscriber interface, as well as N network stations, built on personal computers (PC), providing their interaction.

In wireless local networks with "ad hoc" operating conditions several access protocols can be used but for message exchange in this work the following protocol hereinafter called a controlled access protocol (AP) has been used.

At using a controlled access protocol message by way of packets enters the station synchronously at a fixed interval state T. The transmission of packet-messages is performed by the station in the following sequence. If there's an information packet in the buffer of station 1, it transmits a controlled packet composed of information and service frame to the radio channel in the course of T_{ne} time. Station 2 and all others from N stations receiving this packet don't proceed to the transmission of their packets (if there's one) until transmitting the information frame of station 1, i.e. until the interval T_{ne} is over. If the buffer is empty at the station 1, then it notifies all stations about this (in order to provide cycle synchronization), as well as station 2 about service packet (control order), the transmission time of which is equal to T_e. By this way, if the buffer is occupied (not empty), then that station occupies a radio channel for T_{ne} time, if not occupied (empty) for T_e time. Consequently station 2 gets an access to the radio channel and thereby station N joins the cycle the latest and then the cycle is lost.

In this work an algorithm has been developed and timing diagram of wireless local network performance with a controlled access (WLN with CA) is charted taking into consideration error control with an algorithm critical for feedback and waiting DF – WA.

In correspondence with the algorithm and timing diagram of WLN performance with CA the duration of time window of station transmitting information frame from the occupied buffer has been determined:

$$T_{ne} = t_f + t_{fd} + t_{ac} + 2t_t,\ t_f = n_f V_n^{-1},\ t_{ac} = n_{ac} V_n^{-1},$$

$$t_t = t_{tij} + t_{tji},\quad n_f = k_{pr} + k_f + f + k_o + k_s. \qquad (1)$$

and the duration of time window of station transmitting control order taken from the empty buffer is determined by the following expression:

$$T_e = t_c + t_t + t_{fd},\qquad t_c = n_c V_n^{-1}, \qquad (2)$$

where n_f - frame length, n_{ac} - acknowledge character length, t_{fd} - frame decoding time, t_{tij} - frame transmission time from station i to station j, t_{tji} - transmission time of acknowledge character from station j to station i, V_n – frame transmission rate in radio channel, t_c - transmission time of control order, n_c - control order length, f – length of information part of frame, t_t - transmission time of frame in radio channel, t_f - transmission time of frame in radio channel, t_{ac} - transmission time of acknowledge character, k_{pr} - preambule length, $\mathrm{k_f}$ - field of flags length, $\mathrm{k_o}$ – length of testing order of frame, k_s - service frame length.

In order to develop an analytical model of WLN with CA we'll suppose that Poisson flow of messages comes to the entrance of buffer of its station and its service is realized within discrete time. In this assumption the considered network is modelled by a stochastic system M/G^D/1. Let's suppose that network stations having messages in their buffers in order to transmit the messages are occupied with the probability ρ, and the station buffers not having messages are empty with the probability $\overline{\rho}$.

Base analytical model of WLN taking into account occupied state of its station buffers have the form of [2]:

$$\mathrm{f(s)} = (1-\rho)sg(s)/(s-\lambda(1-g(s))), \tag{3}$$

where s – Laplace operator, g(s) – Laplace transformations for density of distribution of message service interval taken from occupied station buffer of WLN, λ– intensity of message receiving.

Let's determine g(s) for CA protocol. As message service from the occupied station buffer is progressing in integral intervals (discrete time), then let's determine its non-conditional z - transformation in the series of distribution z density of distribution service interval as:

$$\mathrm{g(z)} = g_a(g_c^{-1}(z)), \tag{4}$$

where $g_c(z)$ - z – density of distribution service interval at the first try.

In order to determine $g_c(z)$ let's write down the message service interval at the first attempt as the following expression:

$$n = n_a + \mathrm{n_t}, \tag{5}$$

where n_a is access interval in an occupied station buffer of WLN with CA, and $\mathrm{n_t}$ - message transmission time, which is random quantity.

Taking into consideration those conditions where z – transformation of the sum of two random quantities equals to [3] the production of their z – transformations, the expression (5) will look as:

$$g_c(z) = g_a(\mathrm{z})\mathrm{g_n}(z), \tag{6}$$

where $g_a(\mathrm{z})$ - z - density of distribution of access time at an occupied station buffer of WLN with CA, and $\mathrm{g_n}(z)$ - z – density of distribution of message transmission time taken from an occupied buffer.

Taking into account that $\mathrm{g_n}(z) = z^{n_n}$ the last expression will become like:

$$\mathrm{g_c(z)} = g_a(z)\mathrm{z^{-n_t}},\ \ n_t = n_f + \mathrm{n_{tij}} + n_{ac}, \tag{7}$$

By this way the desired expression for g(s) will be as the following:

$$\mathrm{g(s)} = g(z)\Big|_{z=l^{ST}}, \tag{8}$$

where $g(z)$ is determined by the expression (4) taking into consideration the expression (7) in it.

Let's determine now $g_a(z)$, being a component of the expression (7). For this purpose let's write down the access interval of the message taken from an occupied station buffer of WLN with CA:

$$n_a = \overset{*}{n}_{a_2} + n_{a_3} + \ldots + n_{a_{N-1}}, \tag{9}$$

where $\overset{*}{n}_{a_2}$ - the access interval of the message taken from an empty buffer of the second station, on the transmission of which $\mathrm{T_e}$ time is spent, n_{a_3} - the access interval of the message taken from an empty buffer of the third station, on the transmission of which $\mathrm{T_{ne}}$ time is spent and so on, $n_{a_{N-1}}$ - the access interval of the message taken from an occupied buffer of N-1 station.

On the base of (9) let's put down z – density of distribution of the access interval of i – station within the considered network:

$$g_{a_i}(z) = \rho z^{T_{ne}} + \overline{\rho} z^{-T_e},\ i = 1, \tag{10}$$

and for N-1-station the last expression will be like:

$$g_{a_{N-1}}(z) = (\rho z^{T_{ne}} + \overline{\rho} z^{-T_e})^{N-1}, i = \overline{1, N-1}, \tag{11}$$

Combining the expressions (11) and (7) we can get the final expression for the calculation of the density of distribution for transmitting the message taken from an occupied station buffer of WLN with CA under the terms $z = l^{ST}$, i.e:

$$g_c(s) = (\rho l^{ST\,T_{ne}} + \overline{\rho} l^{-ST\,T_e})^{N-1} - l^{-ST\,n_t}, T = V_n^{-1}. \tag{12}$$

Let's assume that for elimination of the errors generated at message transmission an algorithm critical for feedback and waiting DF – WA is used in the given network. Then the expression (8) for a controlled access protocol taking into account the expressions (4), (7) and (12) in it will look like:

$$g(s) = (Qg_c(s))/(1 - pg_c(s)) \tag{13}$$

$$p = 1 - Q,\quad Q = (1-p)^{n_f},$$

where p is probability of the revealed errors in message, Q is probability of correct message receiving, $g_c(s)$ is determined by the expression (12).

The probability of an occupied state of the station buffer of WLN with CA is defined from interference equation, as:

$$\rho = -\lambda g'(0),\quad g'(0) = \frac{g_c'(0)}{Q}, \tag{14}$$

where $g_c'(0)$ is defined by the way of taking the first derivative from the expression (12) at $s \to 0$, i.e.

$$g_c'(0) = T((1-N)(\rho T_{ne} + \overline{\rho} T_e)\mathrm{n}_{\mathrm{n}}). \tag{15}$$

By this way all components of base analytical model of wireless local network are found out for CA protocol.

Now let's consider the probabilistic-timing performances of WLN with CA evaluating its delay time during information exchange.

The main probabilistic-timing performances are [2]: average delay time $\overline{t}_c$, probability of due message transmission in the network Π_n, information rate of a network of common use R_n^{CU} and real time network R_n^{RT}.

The average delay for the message of the station in the considered network is determined by the following expression:

$$\overline{\mathrm{t}}_c = \left(\frac{d}{ds}\right) f(s)\Big|_{s \to 0}, \tag{16}$$

where $f(s)$ is defined by the expression (3).

Calculating the last expression with taking into consideration the expression (3) after several intermediate transformations we'll get the final expression to calculate the average delay time for the message in WLN with CA:

$$\overline{\mathrm{t}}_c = -g'(0) + (\lambda g''(0)/2(1-\rho)),\ g''(0) = \frac{g_c''(0)}{Q}, \tag{17}$$

where $g'(0)$ is defined by the expression (14) combining the expression (15) in it, $g''(0)$ – is determined with the help of taking the second derivative out of the expression (12) at $s \to 0$, i.e.

$$g''(0) = (g_c''(0)(1-\rho) + 2P(g_c'(0))^2)/(1-\rho)^2 \quad (18).$$

Calculating the second derivatives of (12) at $s \to 0$, after some intermediate transformations, we obtain the calculated expression for $g_c''(0)$ in expression (16):

$$\begin{aligned} g_c''(0) = T^2((N-1)((N-2)(\rho T_{ne} + \overline{\rho}\mathrm{T}_e)^2 + \\ + \rho T_{ne}(T_{ne} + \mathrm{n}_{\mathrm{t}}) + \overline{\rho} T_e(T_e + n_t) + \\ + n_t(\rho T_{ne} + \overline{\rho}\mathrm{T}_e)) + n_t \end{aligned}. \tag{19}$$

Probability of due message transmission in the network is established by the expression (3) changing Laplace operator *s* into a coefficient of message deterioration *v*, i.e.

$$\overline{\Pi}_n = (1-\rho) v g(v)/(v - \lambda(1-g(v))), \tag{20}$$

where $g(v)$ is defined by the expression (13) changing s into v in it, i.e.

$$g(v) = Q g_c(v)/(1-\rho g_c(v)). \tag{21}$$

The information rate for a network of common use to the real time network is correspondingly determined by the following expression:

$$R_n^{CU} = \lambda k N,\quad R_n^{RT} = R_n^{RT}\overline{\Pi}_n, \tag{22}$$

where $\overline{\Pi}_n$ is defined by the expression (19).

On the base of the expressions (17), (18), (19) and (20) taking into account in it (1), (2), (14), (15) and (18) and the parameters of the networks under consideration by the way of calculating experiment integral values of probabilistic-timing characteristics of these networks have been defined for a controlled access and their comparative analysis has been carried out.

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