Numerical Modeling of Development of Oil Deposit with System of Wells Taking into Account the Relaxation of Deformation of Rocks

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Abstract— **The paper presents a numerical model for solving the task of designing a system of randomly located wells oil field, which deformed rocks with relaxation.**

Keywords— relaxation; pressure; porosity; oil saturation; gas-oil ratio

Many developed deep oil and gas fields are characterized by abnormally high reservoir pressures and high temperatures and oil and gas bearing rock at such fields under huge geostatic pressure are exposed to strong and frequently inelastic deformation. The complexity of geological structure, fracturing, cavernosity and occurrence of shale and salt in such field intensify inelastic - relaxation and creeping rock deformation.

It is necessary to note that nowadays at predevelopment and development stage at many of deep fields of Azerbaijan, Northern Caucasus, Central Asia and especially in the Pre-Caspian Region it is necessary to take into account relaxation and creeping deformation of oil and gas saturated rock of productive formations and overlying and underlying rock whenever possible. There are many laboratory experiment data the confirm inelastic, relaxation and creeping behavior of reservoirs of oil and gas [1,2,3,5]. Also there is information about inelastic behavior of field rock during development. There is general understanding of filtration of fluids in formations that are characterized by different rheological conditions. At the same time the scope of problems that are resolved is limited. It does not allow to quantify the mechanism of impact of different rheological properties of rock on field development process and to calculate reservoir management processes taking into account primarily relaxation and creeping deformation of rock, definitively and accurately interpret results of formation and well tests in rheological conditions, to analyze development of oil and gas fields in these conditions.

The first studies of filtration of liquid in the porous medium were conducted with the assumptions of nondeformability of rock composing oil and gas reservoir and noncompressibility of liquid contained in pores of this rock. In accordance with these assumptions bottomhole pressure after well shut-in should build up immediately because in this case porous medium during filtration (due to formation pressure decline or increase) does not deform, i.e. relative deformation equals to zero and the system is absolutely rigid. However, these tests of oil and gas wells showed that with any change of well flow rates the formation pressure both in gas and in oil reservoirs builds up gradually over a certain time and not immediately which was explained by compressibility of rock and fluids.

The paper [3] provides rheological models for description of rock deformation and known relationships of rock porosity and permeability and pressure and time in the conditions of elastic and inelastic deformation. It describes the solutions of the problems of oil and gas filtration in formations with relaxation and creeping medium obtained by the authors and methods of interpretation of results of well tests for determination of reservoir properties and relaxation and creeping properties. The paper describes the results of modeling of development of depletion driven oil, gas and oil and gas fields taking into account relaxation and creeping of rock. The authors discuss further development of studies related to creation of methods of calculation of reservoir management indicators and determination of reservoir and rheological properties of deposits hosted in inelastically deforming rock.

The Institute of Geology of Azerbaijan National Academy of Sciences have studied the impact of relaxation and creeping deformation of rock on process parameters of development of oil and gas field and determination of their reservoir properties and rheological parameters. The Institute have also studied application of these parameters for estimation of hydrocarbon resources, etc.

In the paper a numerical model for solving the task of designing a system of randomly located wells oil field is presented, which deformed rocks with relaxation. The mode of the dissolved gas considered the problem is resolved as follows:

$$\frac{\partial}{\partial x}\left(\frac{\bar{k}_{oil}(\sigma)}{\mu_{oil}(p)a(p)}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\bar{k}_{oil}(\sigma)}{\mu_{oil}(p)a(p)}\frac{\partial p}{\partial y}\right)=\frac{1}{k_0}\frac{\partial}{\partial t}\left(\frac{m}{a(p)}\sigma\right)+$$
$$+\frac{1}{h\cdot k_0}\sum_{\nu=1}^{N_1} q_{\nu\, oil}(t)\cdot\delta(x-x_\nu)\delta(y-y_\nu)$$

$$\frac{\partial}{\partial x}\left\{\left(\frac{\overline{k}_{oil}(\sigma)s(p)}{\mu_{oil}(p)a(p)}+\frac{\beta}{p_{at}}\frac{\overline{k}_{gas}(\sigma)p}{\mu_{gas}(p)}\right)\frac{\partial p}{\partial x}\right\}+$$
$$+\frac{\partial}{\partial y}\left\{\left(\frac{\overline{k}_{oil}(\sigma)s(p)}{\mu_{oil}(p)a(p)}+\frac{\beta}{p_{at}}\frac{\overline{k}_{gas}(\sigma)p}{\mu_{gas}(p)}\right)\frac{\partial p}{\partial y}\right\}=$$
$$=\frac{1}{k}\frac{\partial}{\partial t}\left\{m\left(\frac{s(p)}{a(p)}\sigma+\frac{\beta}{p_{at}}p(1-\sigma)\right)\right\}+$$
$$+\frac{1}{h\cdot k_0}\sum_{\nu=1}^{N_1} q_{\nu\, oil}(t)\cdot\Gamma\cdot\delta(x-x_\nu)\delta(y-y_\nu)$$

$$m+\tau_m\frac{\partial m}{\partial t}=m_0 e^{\beta_c(p-p_0)}$$

$$p(x,y,0)=p_0\,,\quad m(x,y,0)=m_0\,,\quad \sigma(x,y,0)=1$$

$$\left.\frac{\partial p}{\partial x}\right|_{x=0;L}=0\,,\quad \left.\frac{\partial p}{\partial y}\right|_{y=0;H}=0$$

where $\Gamma=\frac{\beta}{p_{at}}\cdot\frac{\overline{k}_{gas}(\sigma)}{\overline{k}_{oil}(\sigma)}\cdot\frac{\mu_{oil}(\sigma)}{\mu_{gas}(\sigma)}a(p)p+s(p)$; $p=p(x,y,t)$, $m=m(x,y,t)$, $\sigma=\sigma(x,y,t)$; $0\le x\le L$, $0\le y\le H$, $t\ge 0$; p , m and σ - current reservoir pressure, current porosity and oil saturation, respectively; p_0 and m_0 - initial reservoir pressure and initial porosity, respectively; N_1 - number of wells; H , L and h - width, length and thickness of strip-shaped deposit; $q_{\nu\, oil}(t)$ - oil flow rate of well ν; $\overline{k}_{oil}$ and $\overline{k}_{gas}$ - oil and gas phase permeability, respectively; k_0 - absolute permeability of reservoir; $a(p)$ and $s(p)$ - formation volume factor and gas-in-oil solubility factor; μ_{oil} and μ_{gas} - viscosity of oil and gas, respectively; δ - Diract function; β - temperature adjustment for gas; p_{at} - atmospheric pressure; τ_m - porosity relaxation time; β_c - formation rock compressibility coefficient.

It is necessary to replace its integration domain (based on variables x , y and t) by a respective discrete and node domain to find numerical solutions of the problem

$$\omega=\{(x_i,y_j,t_k): x_i=i\cdot\Delta x\ (i=0,1,...,N; N=L/\Delta x);$$
$$y_j=j\cdot\Delta y\ (j=0,1,...,M; M=H/\Delta y); t_k=k\cdot\tau, k=0,1,2,...\}.$$

The following iteration expressions are obtained by approximation of original differential equations and initial and marginal conditions with their respective difference analogs for determination of pressure, porosity and oil saturation in net domain nodes with indices (i,j,k) [4]:

- for pressure

$$p_{i,j}^{k,r}=\frac{1}{a_{i,j}^{k,r}}\left(d_{i,j}^{k}+g_{i,j}^{k,r}\cdot p_{i,j-1}^{k-1}+c_{i,j}^{k,r}\cdot p_{i-1,j}^{k-1}+b_{i,j}^{k,r}\cdot p_{i+1,j}^{k-1}+e_{i,j}^{k,r}\cdot p_{i,j+1}^{k-1}\right)$$

- for porosity

$$m_{i,j}^{k}=m_{i,j}^{k-1}\left(1-\frac{\tau}{\tau_m}\right)+\frac{\tau}{\tau_m}m_0 e^{\beta_c(p_{i,j}^{k-1}-p_0)}$$

- for oil saturation

$$\sigma_{i,j}^{k,r}=\sigma_{i,j}^{k-1}+\frac{\Phi_{4,i,j}^{k-1}}{\Phi_{2,i,j}^{k-1}}\frac{k_0\tau}{(\Delta x)^2}\left[F_{1,i+1/2,j}^{k,r-1}\left(p_{i+1,j}^{k-1}-p_{i,j}^{k,r}\right)-F_{1,i-1/2,j}^{k,r-1}\left(p_{i,j}^{k,r}-p_{i-1,j}^{k-1}\right)\right]+$$
$$+\frac{\Phi_{4,i,j}^{k-1}}{\Phi_{2,i,j}^{k-1}}\frac{k_0\tau}{(\Delta y)^2}\left[F_{1,i,j+1/2}^{k,r-1}\left(p_{i,j+1}^{k-1}-p_{i,j}^{k,r}\right)-F_{1,i,j-1/2}^{k,r-1}\left(p_{i,j}^{k,r}-p_{i,j-1}^{k-1}\right)\right]+$$
$$+\frac{\Phi_{3,i,j}^{k-1}}{\Phi_{2,i,j}^{k-1}}\frac{k_0\tau}{(\Delta x)^2}\left[F_{2,i+1/2,j}^{k,r-1}\left(p_{i+1,j}^{k-1}-p_{i,j}^{k,r}\right)-F_{2,i-1/2,j}^{k,r-1}\left(p_{i,j}^{k,r}-p_{i-1,j}^{k-1}\right)\right]+$$
$$+\frac{\Phi_{3,i,j}^{k-1}}{\Phi_{2,i,j}^{k-1}}\frac{k_0\tau}{(\Delta y)^2}\left[F_{2,i,j+1/2}^{k,r-1}\left(p_{i,j+1}^{k-1}-p_{i,j}^{k,r}\right)-F_{2,i,j-1/2}^{k,r-1}\left(p_{i,j}^{k,r}-p_{i,j-1}^{k-1}\right)\right]+$$
$$+\frac{\beta}{p_{at}}\frac{\Phi_{3,i,j}^{k-1}}{\Phi_{2,i,j}^{k-1}}\frac{k_0\tau}{(\Delta y)^2}\left[F_{3,i,j+1/2}^{k,r-1}\left(p_{i,j+1}^{k-1}-p_{i,j}^{k,r}\right)-F_{3,i,j-1/2}^{k,r-1}\left(p_{i,j}^{k,r}-p_{i,j-1}^{k-1}\right)\right]-$$
$$-\frac{\tau}{\Phi_{2,i,j}^{k-1}}\left(\Psi_{3,i,j}^{k-1}-\Phi_{3,i,j}^{k-1}\left[\Psi_{1,i,j}^{k-1}+\Psi_{2,i,j}^{k-1}\cdot\frac{1}{h}\sum q_{\nu\, oil}(t_{k-1})\delta(x_i-x_\nu)\delta(y_j-y_\nu)\right]\right)$$

$$i=\overline{1,N-1}\,,\ j=\overline{1,M-1}\,,\ k=1,2,..\,,\ r=1,2,..,$$

$$p_{i,j}^{0}=p_0\,,\ m_{i,j}^{0}=m_0\,,\ \sigma_{i,j}^{0}=1\,,\ i=\overline{0,N}\,,\ j=\overline{0,M}$$

$$p_{0,j}^{k,r}=p_{1,j}^{k,r}\,,\ p_{N,j}^{k,r}=p_{N-1,j}^{k,r}\,,\ p_{i,0}^{k,r}=p_{i,1}^{k,r}\,,\ p_{i,M}^{k,r}=p_{i,M-1}^{k,r}\,,$$

$$i=\overline{1,N-1}\,,\ j=\overline{1,M-1}$$

where

$$a_{i,j}^{k,r}=\frac{\Phi_{1i,j}^{k-1}}{\tau}+\frac{k_0}{(\Delta x)^2}F_{0,i,j}^{k-1}\left[F_{1,i+1/2,j}^{k,r-1}+F_{1,i-1/2,j}^{k,r-1}\right]+\frac{k_0}{(\Delta y)^2}F_{0,i,j}^{k-1}\left[F_{1,i,j+1/2}^{k,r-1}+F_{1,i,j-1/2}^{k,r-1}\right]+$$
$$+\frac{k_0}{(\Delta x)^2}\left[F_{2,i+1/2,j}^{k,r-1}+F_{2,i-1/2,j}^{k,r-1}\right]+k_0\frac{\beta}{p_{at}}\frac{1}{(\Delta x)^2}\left[F_{3,i+1/2,j}^{k,r-1}+F_{3,i-1/2,j}^{k,r-1}\right]+$$
$$+\frac{k_0}{(\Delta y)^2}\left[F_{2,i,j+1/2}^{k,r-1}+F_{2,i,j-1/2}^{k,r-1}\right]+k_0\frac{\beta}{p_{at}}\frac{1}{(\Delta y)^2}\left[F_{3,i,j+1/2}^{k,r-1}+F_{3,i,j-1/2}^{k,r-1}\right];$$

$$d_{i,j}^{k}=\frac{\Phi_{1,i,j}^{k-1}\cdot p_{i,j}^{k-1}}{\tau}-\Psi_{1,i,j}^{k-1}--\frac{\Psi_{2,i,j}^{k-1}}{k_0\cdot h}\sum_{\nu=1}^{N_1}q_{\nu н}(t_{k-1})\delta(x_i-x_\nu)\delta(y_j-y_\nu)$$

$$g_{i,j}^{k,r}=\frac{k_0}{(\Delta y)^2}F_{0,i,j}^{k-1}F_{1,i,j-1/2}^{k,r-1}+\frac{k_0}{(\Delta y)^2}F_{2,i,j-1/2}^{k,r-1}+\frac{\beta}{p_{at}}\frac{k_0}{(\Delta y)^2}F_{3,i,j-1/2}^{k,r-1}\,,$$

$$c_{i,j}^{k,r}=\frac{k_0}{(\Delta x)^2}F_{0,i,j}^{k-1}F_{1,i-1/2,j}^{k,r-1}+\frac{k_0}{(\Delta x)^2}F_{2,i-1/2,j}^{k,r-1}+\frac{\beta}{p_{at}}\frac{k_0}{(\Delta x)^2}F_{3,i-1/2,j}^{k,r-1},$$

$$b_{i,j}^{k,r}=\frac{k_0}{(\Delta x)^2}F_{0,i,j}^{k-1}F_{1,i+1/2,j}^{k,r-1}+\frac{k_0}{(\Delta x)^2}F_{2,i+1/2,j}^{k,r-1}+\frac{\beta}{p_{at}}\frac{k_0}{(\Delta x)^2}F_{3,i+1/2,j}^{k,r-1},$$

$$e_{i,j}^{k,r}=\frac{k_0}{(\Delta y)^2}F_{0,i,j}^{k-1}F_{1,i,j+1/2}^{k,r-1}+\frac{k_0}{(\Delta y)^2}F_{2,i,j+1/2}^{k,r-1}+\frac{\beta}{p_{at}}\frac{k_0}{(\Delta y)^2}F_{3,i,j+1/2}^{k,r-1},$$

$$\delta(x_i-x_\nu)\delta(y_j-y_\nu)=\begin{cases}0, & x_i\neq x_\nu,\ y_j\neq y_\nu\\ \dfrac{1}{\Delta x\Delta y}. & x_i=x_\nu,\ y_j=y_\nu\end{cases}$$

$$\Phi_1=\left(s'(p)-\frac{\beta}{p_{at}}\left(a(p)+p\cdot a'(p)\right)\right)\frac{m\sigma}{a(p)}+\frac{\beta}{p_{at}}m,$$

$$\Phi_2=\frac{m}{a(p)},\quad \Phi_3=\frac{m\sigma}{a^2(p)}a'(p)\frac{1}{\Phi_1},\quad \Phi_4=1+\frac{m\sigma}{a^2(p)}a'(p)\frac{F}{\Phi_1},$$

$$\Psi_1=\frac{\beta}{p_{at}}p\frac{1}{\tau_m}\left(m_0e^{\beta_c(p-p_0)}-m\right),\quad \Psi_2=\frac{\beta}{p_{at}}p\cdot a(p)\left(\frac{\bar{k}_{gas}}{\bar{k}_{oil}}\cdot\frac{\mu_{oil}}{\mu_{gas}}+1\right),$$

$$\Psi_3=\frac{\sigma}{a(p)}\frac{1}{\tau_m}\left(m_0e^{\beta_c(p-p_0)}-m\right),\ F_0=\frac{\beta}{p_{am}}p\cdot a(p)-s(p),$$

$$F_1=\frac{\bar{k}_{oil}(\sigma)}{\mu_{oil}(p)\cdot a(p)},\ F_2=\frac{\bar{k}_{oil}(\sigma)\cdot s(p)}{\mu_{oil}(p)\cdot a(p)},\ F_3=\frac{\bar{k}_{gas}(\sigma)\cdot p}{\mu_{gas}(p)},$$

$$p_{i-1/2,j}^{k,r-1}=\frac{2p_{i,j}^{k,r-1}p_{i-1,j}^{k,r-1}}{p_{i,j}^{k,r-1}+p_{i-1,j}^{k,r-1}},\quad p_{i+1/2,j}^{k,r-1}=\frac{2p_{i,j}^{k,r-1}p_{i+1,j}^{k,r-1}}{p_{i,j}^{k,r-1}+p_{i+1,j}^{k,r-1}},$$

$$p_{i,j-1/2}^{k,r-1}=\frac{2p_{i,j}^{k,r-1}p_{i,j-1}^{k,r-1}}{p_{i,j}^{k,r-1}+p_{i,j-1}^{k,r-1}},\quad p_{i,j+1/2}^{k,r-1}=\frac{2p_{i,j}^{k,r-1}p_{i,j+1}^{k,r-1}}{p_{i,j}^{k,r-1}+p_{i,j+1}^{k,r-1}},$$

$$p_{i\pm1,j}^{k,r-1}=p_{i\pm1,j}^{k-1},\ p_{i,j\pm1}^{k,r-1}=p_{i,j\pm1}^{k-1}$$

r is number of iteration for each time layer t_k $(k=1,2,...)$; Δx, Δy and τ are steps of integration based on variables x, y and t, respectively.

Iteration process continues based on obtained iteration ratios for each k time layer until required accuracy is achieved, i.e. until the inequality is satisfied

$$\left|p_{i,j}^{k,r}-p_{i,j}^{k,r-1}\right|\le\varepsilon$$

where ε is required calculation accuracy. In each iteration $p_{i,j}^{k,r}$, $r=1,2,...$ it is first assumed that $p_{i\pm1,j}^{k,0}=p_{i\pm1,j}^{k-1}$, $p_{i,j}^{k,0}=p_{i,j}^{k-1}$, $p_{i,j\pm1}^{k,0}=p_{i,j\pm1}^{k-1}$ (with $k=1$: $p_{i,j}^0=p_0$, $m_{i,j}^0=m_0$, $\sigma_{i,j}^0=1$).

At the same time porosity in each time layer in accordance with provided iteration is determined only by application of pressure and porosity values from the previous time layer.

In such solution in nonlinear value of preceding iteration r -1, i.e. values $p_{i,j}^{k,r-1}$, are taken in nonlinear coefficients instead of values $p_{i,j}^{k,r}$. And the requirement that inequality $\left|p_{i,j}^{k,r}-p_{i,j}^{k,r-1}\right|\le\varepsilon$ should be satisfied for each time layer t_k ensures adjustment of these coefficients for the same time layers [6].

It is necessary to note that in nodes x_ν, y_ν, t_k calculated values of pressure and oil saturation will not correspond to their actual values. For each time value using these values it is possible to calculate bottomhole pressure and oil saturation using appropriate equation for stabilized flow of gas-cut oil in well in elementary cylindrical volume with radius $r_k=0{,}2077\Delta x$ (with $\Delta x=\Delta y$). Pressure values calculated in nodes x_ν, y_ν, t_k are assumed as mean weighted pressures in this elementary volume [6].

Gas-oil ratio for calculation of pressure and oil saturation in different points is calculated in these points using the following equation:

$$\Gamma=\frac{\beta}{p_{at}}\cdot\frac{\bar{k}_{gas}(\sigma)}{\bar{k}_{oil}(\sigma)}\cdot\frac{\mu_{oil}(\sigma)}{\mu_{gas}(\sigma)}a(p)p+s(p).$$

Calculation for an uniform well spacing with the following input data is provided as an example:

$p_0=40\cdot10^6$ Pa; $N_1=9$; $q_1=q_2=...=q_{N_1-1}=q_{N_1}=50$ m^3/day;

$m_0=0{,}2$; $h=10$ m; $H=400$ m; $L=700$ m; $r_c=0{,}1$ m;

$\beta=0{,}735$; $z(p_0)=1{,}0385$; $p_{at}=10^5$ Pa;

$\beta_c=2{,}5\cdot10^{-9}$ Pa^{-1}; $\tau_m=0$; $\tau_m=5$ years;

$\mu_{oil}=\mu_{oil\,0}\exp(-0{,}005(p-p_0))$; $k_0=0{,}1\cdot10^{-12}$ m^2;

$\mu_{oil\,0}=0{,}5$, in centipoise; $\bar{k}_{gas}=1{,}16(1-\sigma)^2$;

$\bar{k}_{oil}=1{,}06\cdot\sigma^3-0{,}06$;

$\mu_{gas}(p)=0{,}0054(p/p_0)^2+0{,}0114(p/p_0)+0{,}0105$,

in centiopoise;

$z(p)=-0{,}1845(p/p_0)^3+0{,}9458(p/p_0)-0{,}72(p/p_0)+0{,}9972$;

$$s(p)=\begin{cases}3{,}9\cdot p+15{,}5, & p\ge5\cdot10^6\ Pa\\ 4\cdot p, & p<5\cdot10^6\ Pa\end{cases};$$

$$a(p)=\begin{cases}0{,}0058\cdot p+1{,}021, & p\ge5\cdot10^6\ Pa\\ 0{,}01\cdot p+1, & p<5\cdot10^6\ Pa\end{cases}$$

Bottomhole and contour values of pressure, oil saturation and gas-oil ratio in nodal points of the grid area of the deposit were determined by calculation. Bottomhole pressure, oil saturation and gas-oil ratio curves in oil deposit with nonlinear-elastic and relaxation rock deformation for a center wells of a well pattern are shown as an example on Fig.1-3, respectively.

The analysis of these curves shows that in the considered case when oil recovery factor of 0,176 is reached (it corresponds to development time of 219 days) in case of relaxation deformation of rock the value of bottomhole pressure and oil saturation is significantly lower and the value of gas-oil ratio is significantly higher than respective values in case of nonlinear-elastic rock deformation (44,8%, 9,7% and 57,2%, respectively).

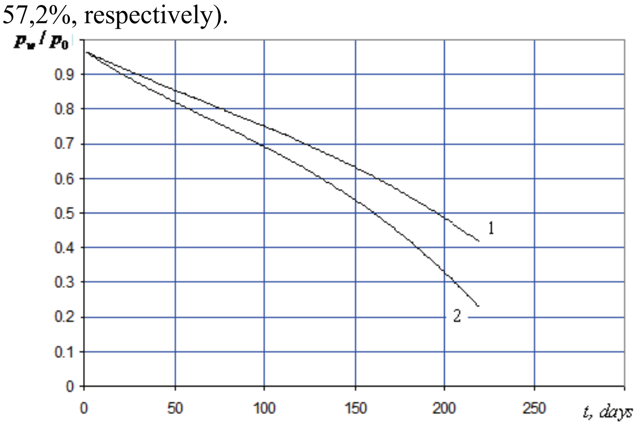

Figure 1. Variation of bottomhole pressure in a central well of an oil deposit in case of nonlinear-elastic (1) and relaxation (2) rock deformation

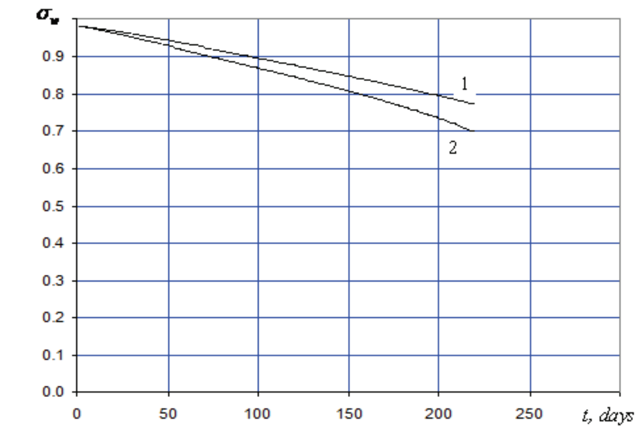

Figure 2. Variation of bottomhole oil saturation in an oil deposit in case of a nonlinear-elastic (1) and a relaxation (2) rock deformation

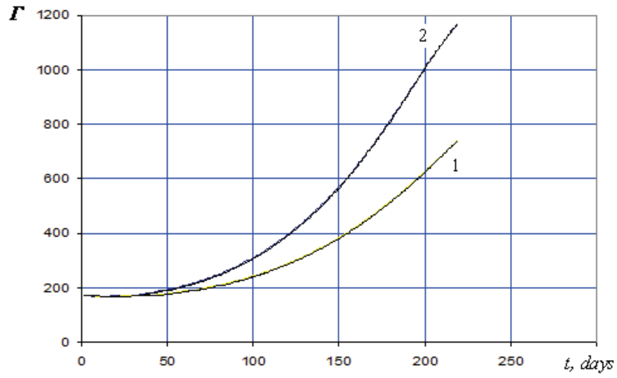

Figure 3. Variation of bottomhole gas-oil ratio in an oil deposit in case of a nonlinear-elastic (1) and a relaxation (2) rock deformation

The above curves shows that relaxation rock deformation in the considered case (with defined well flow rate) relative to nonlinear-elastic deformation has a significant impact on the variation of studied petrophysical and process characteristics of an oil deposit when it is developed with a dissolved gas drive.

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