On the Separability of the Parameter Space of Dynamical Systems in Terms of Types of Phase Portraits

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***Abstract*— We investigate the question of the separability of the space of input parameters of industrial reaction-regeneration systems in terms of topological structures of the phase trajectories. The phase trajectories are investigated in the vicinity of the set of attractors. We propose a method for fixing the conformity for each input vector of the 3D phase portrait of a particular system. Results of the decision of a problem of visualisation of the control with use of phase portraits of nonlinear dynamic systems are presented.**

Keywords*— nonlinear dynamics of system with multiplicity of attractors; 3D phase portrait, approximation of phase space of nonlinear systems; bifurcation of solutions

I. INTRODUCTION

Given a nonlinear dynamical system

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}); \quad \boldsymbol{x} = \{x_1, x_2, x_3\}; \boldsymbol{u} = \{u_1, u_2\};$$

$$\boldsymbol{f} = \{f_1, f_2, f_3\}; t > 0 \quad ,$$

which can be characterized by multiple equilibrium $\boldsymbol{x}_j^* = \{x_{j1}^*, x_{j2}^*, x_{j3}^*\}; j = \overline{1,3}$. These equilibrium states are determined by conditions $f_i(\boldsymbol{x}_j^*, \boldsymbol{u}) = 0; i = \overline{1,3}$ generally refer to different types of topological forms of organization with the appropriate flow paths in their neighborhoods. Working out of this or that strategy of management with use of phase portraits is complicated by that influence of a vector of management on them in a qualitative sense takes place. The preliminary analysis possible bifurcations decision in connection with variations of a vector of managements is necessary. It is necessary to allocate areas of qualitatively same phase portraits. Approximation of the given computing experiments on revealing of quantitative influence of a vector of management on phase portraits within the allocated areas further should follow. The characteristics received thus can form a basis for synthesis of laws in managements.

Analysis of the topology of two-dimensional manifolds with the use of computer technology is based on the principles of the theory of the Poincare-Bendixson [1]. Note that for problems of control real physical objects because of the limitations of the real physical states is not necessarily conduct a qualitative analysis of solutions in the whole phase space. This is a consequence of the fact that the models of real objects, particularly objects carrying industrial processes, can claim only the adequacy of the areas of nominal modes of operation. Computer construction of Poincare sections, reflecting the behavior of dynamic models in an infinite domain of the state variables, which have recently noticed some interest in connection with a number of problems of nonlinear dynamics [2,3], in this case is not only unnecessary, but complicating the synthesis and control functions. Another problem associated with the dimension of controlled dynamic systems. The transition to systems with three degrees of freedom requires the solution of a considerable number of algorithmic problems associated with visualization of the trajectories. In determining the control strategy more rationally assume it is the sufficiency of the fragments in AF control systems, which characterize the nominal ("working") of state G variables.

II. SEPARABLE SPACE OF THE THERMAL MODEL OF REACTION-REGENERATION SYSTEM FOR PETROLEUM CRACKING

The complex tasks associated with managing dynamic systems with a multiplicity of equilibrium based on the consideration of 3D slice AF, it is easier to identify on the example of a specific system. Consider the 3D structure of the phase space of one particular model - from the perspective of the local roundedness of the states described by the nominal and the other quite global in the sense that for this model, the nominal area of the phase states characterized by a very complex organization flow trajectories in the neighborhood of a stationary solution is not - a few. This model describes the thermal regimes of a closed system [4], carrying out controlled catalytic reaction and the regeneration of spent catalyst, which is a well-studied from the viewpoint of topology, making the following equations:

$$\frac{dx_1}{dt} = 0.6(x_3 - x_1) - 2.1\, x_1 \exp\left(-0.3\frac{1-x_2}{x_2}\right);$$
$$\frac{dx_2}{dt} = c_1 - c_2;\; c_1 = 0.11x_1 \exp\left(0.3\frac{1-x_2}{x_2}\right) + 0.16u_1;$$
$$c_2 = (0.23 + u_2u_1)x_2 + 0.37 \qquad (1)$$
$$\frac{dx_3}{dt} = 10u_1(x_1 - x_3) + 100u_1^2(th(2.4x_2 - 2.4) + 2);$$

This system is for certain values of the parameters u_1, u_2 can have one, two, three stationary solutions belonging to the area is physically possible states G, or it does not have any solutions. These solutions correspond to the condition $\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dx_3}{dt} = 0$. We denote the singular points of the phase space corresponding to these solutions by $x^*_{j1}, x^*_{j2}, x^*_{j3};\ j = \overline{1,3}$. The singular point of the phase space x^*_{j2} is a type of "saddle" (Figure 1a, b), through which the surface-to separator separating the streams of the trajectories, forward to the two centers of gravity x^*_{j1}, x^*_{j3}. 3D phase portrait of system (1) in the presence of two stable equilibrium (attractors) is shown in Figure 1. This is one of the options is qualitatively different types of phase portraits of system (1).

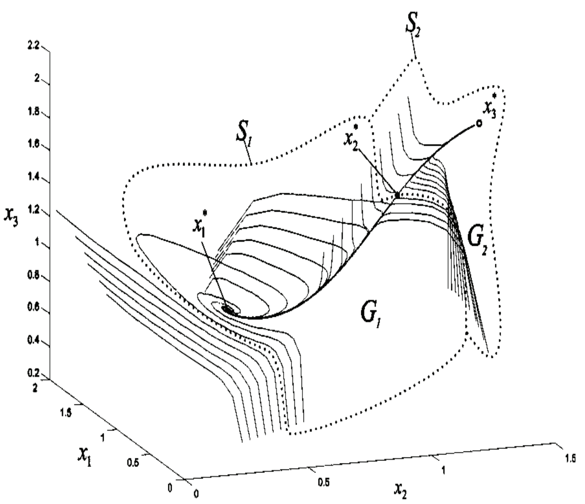

Figure 1. The phase portrait of the system when there are two attractors with domains of attraction G_1, and G_2 their separating surfaces S_1 and S_2.

The total number found for this system is qualitatively different phase portraits comes to nine. The values of the parameters u_1, u_2, causing alteration in the qualitative phase portraits of well-equipped area. Moreover, issues of great importance, is the convexity of these local areas.

Display of area of admissible managements to a numerical axis ξ by means of function $\xi = u_1u_2$ forms five convex subsets:

$$U = \{u_{k\min} \le u_k \le u_{k\max}; k = 1,2\}$$
$$b_\ell < \xi(\boldsymbol{u}) < b_{\ell+1};\ \ell = \overline{1,5}.$$

Figure 2 shows this mapping.

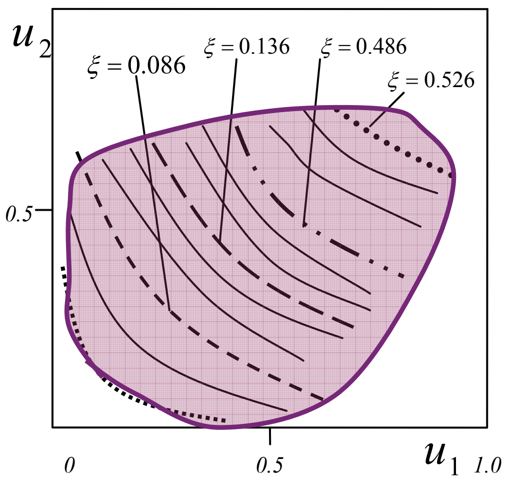

Figure 2. Sub domain is qualitatively similar solutions of (1) of subdivision $\xi = u_1u_2$.

A convex partition of the parameter ξ, which is of great importance for studying the properties of this model, however, little is expedient in control, as a function of $\xi(u) = u_1u_2$ lacks the independence of these offices. More efficient method is suggested below.

III. Approximation Of Computer Experiments

The basis of this method is supposed to end with the approach of a given set of inputs within dynamic systems, and creating a database for grouping 3D phase portraits by attributes. Establishment of phase portraits, the formation of groups on the basis of the qualitative structure of space, as well as fixing the match "entry - the phase portrait" is an integral part of the phase space approximation of nonlinear systems. Figure 3 shows a diagram illustrating the principle of grouping by attributes. No small role in the functioning of this algorithm is the systematization of the phase portraits of the vectors $\{u_{1i}, u_{2i}\}; i = 1,2,3,\ldots,N$, is the principle of collecting requisite maps (Fig. 4). Maps contain all the requisite information about the properties of phase portraits, including the beginning of the trajectories during the transition from initial states to the endpoint of the trajectory, etc.

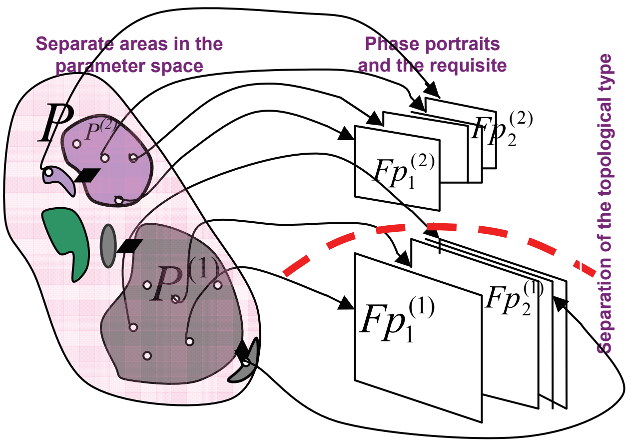

Figure 3. One mapping of finite subsets of $P^{(i)}$ in the subset of phase portraits of $G^{(i)}$.

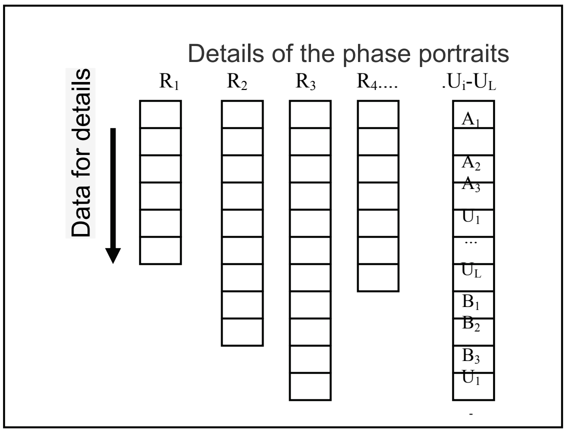

Figure 4. Requisite for a map of the phase portrait.

The basic information material requisite maps the vectors - the nodes of piecewise linear approximations to the solutions of (1). In Fig. 4, these vectors are denoted as (Ui-UL). Before record of each such vector the numerical information is placed. They represent the following estimates:

- A1-the number of nodal points of the trajectory;
- A2-transition time from the beginning of the trajectory to its end;
- A3-coordinate of a point fixes the intersection of the attractor.

This same information is stored in relation to other attractors, which are denoted in Figure 3 $(B)_i$:

- R-requisites are the following:
- R1-coordinates of the attractors and saddle points of equilibrium;
- R2-the number of trajectories that fill the basin of attraction of the attractor;
- R3-value of controls that give rise to this phase portrait;
- R4-approved number of topological type.

Such a data archiving, preservation and retrieval of phase portraits in line with the control vector is called Multiple-phase portrait of dynamical system.

Each vector (u_1, u_2), which is used in the process of visual control is designed on the basis of visual representations associated with a particular phase portrait shown on the monitor. The approximation of phase portraits of themselves and the surfaces separating the attractors S_1 and S_2 carried out by fixing the trajectories in a spherical coordinate system and triangulations of closed surfaces of separation. It is proposed a special rule of triangulation, using a pattern evenly distributed over the spherical surface of radius points. We study algorithms for the construction of such a spherical pattern and a triangulation scheme for latitude and longitude coordinates. We show the effectiveness of the triangulation when compared with the conventional algorithms based on Delaunay method [5].

The notion of "multibay" phase portrait, characterized in that the data are taken not as an archived matrix of solutions of differential equations, as calculated by a predetermined approximations computing experiments to cover the separated areas of the input data $P^{(i)}; i = \overline{1,N}$:

$$X_j^* = X_j^*(p); \quad j = \overline{1,2}; \quad p \in P$$

where $X_j^*; \quad j = \overline{1,2};$ are vectors of the centers of gravity (the coordinates of the attractors); p is the vector of parameters input. On the other hand the approximation of the radius vectors of spherical coordinates in the areas of $P^{(i)} = \{p_1, p_2, ..., p_m\}; i = \overline{1,N}$, allows for an arbitrary input vector to construct the corresponding dividing surface.

The complex called the type of approximation provides the basis for the visualization of phase control, using the coordinates of nonlinear systems with multiple attractors.

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