Modeling of Appearance of Instability of Complex Systems

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Abstract— **The main objective of the work is to strengthen the capacity in study of uncertainty impact on security of complex systems for which standard modeling methods are inadequate. The regimes with sudden, discontinuous changes that cause the instability have been modeling for a wide range of different complex systems from economics and ecology to sociology and biology.**

Keywords— modeling; complex systems; theory of catastrophes

I. Introduction

Recently, one can observed the dramatic increase of the frequency of rare events and their destructive power. There are a sharp increasing of the number of natural and man-made disasters, financial and economical crises. The unique nature of rare events does not completely allow the correct use of probability theory to assess the risk posed by the crisis management systems, reduces the time level of extreme events, especially related with the power-law probability density distribution of the damages which decrease more slowly than a Gaussian distribution law of probability. Analysis of rare events shows that usually they are related with sudden jumping changes, following from continuous low impact on the system.

Traditional methods of prediction of their behavior are not sufficiently efficient for similar events. New risks are requiring new research methods. Thus, the necessity of the work is related by the appearance of new forms of global issues connected with a wide range of asymmetric threats, high sensitivity to initial conditions, and increase of the possibility that a small disturbance can lead to unpredictable ecological, economical, epidemiological, social and political consequences.

II. Modeling of Mechanisms of Instability of Complex System

Modeling methodology is based on a mathematical formalism for modeling of nonlinear systems whose behavior shows sudden, discontinuous changes or phase transitions, resulting from small continuous changes in variables which are affect to the system - on the theory of catastrophes (TC), which was applied to a wide range of various systems such as physical, engineering, biological, psychological and sociological systems. A small list of specific phenomena, which are analyzed and modeled using the TC includes quantum morpho genesis, the formation of caustics in the ray optics, the stability of black holes, morphogenesis, perceptual bi stability [1].

The methodology of risk assessment for sudden transition from one steady state to another, based on the use of TC is considered in [2-4]. It allows the calculation of the bifurcation values, curves and surfaces of the control parameters. The transition probability is estimated as a measure of the approximation of values of control parameters to their bifurcation values, which characterize the system's transition from one stationary state (normal) to another (catastrophe).

Assume that a complex system satisfies all requirements of the potential system and can be described by a potential function $U(x, A_1, A_2, \ldots A_N)$ of the behavioral variable x and the control parameters A_i. The dynamics of deterministic gradient system is described by the equation of the form

$$dx/dt = -\partial U/\partial x \qquad (1)$$

Eq. (1) means that the variable is changed in the direction of decreasing of the potential at a rate proportional to the slope of the gradient of the potential field. Equilibrium manifold of the system is a set of variables x such that $dx/dt = -\partial U/\partial x = 0$.

For example, if

$$U(x, A_1, A_2, A_3, A_4, A_5) = 1/6x^6 + 1/4A_1x^4 + $$
$$+1/3A_2x^3 + 1/2A_3x^2 + A_4x + A_5, \qquad (2)$$

then, the system is in equilibrium when $x^5 + A_1x^3 + A_2x^2 + A_3x + A_4 = 0$.

The function U has five stationary states: three of them are stable whereas 2 others are unstable. Moving the system from one stationary state to another or a change in the nature of the stationary state (for example, from a stable to an unstable) is a function of the control parameters A. These parameters control both the movement of the image point on the surface U and the transformation of this surface.

The number of stationary states of the system can be determined by analyzing a set of input data. Let's assume this analysis shows that the investigated complex system has three stable stationary states. For simplicity, one can make the following assumptions. The first stable stationary state characterizes normal conditions. The level in this state is minimal. The second one characterizes the state with average x.

The third one describes the crisis with a high level x. Under these assumptions, the potential function U is described by eq. (2).

This case with three stable stationary states and the four control parameters corresponds to one of the universal deformation theory of catastrophes, which is called as "butterfly":

$$-\partial U(x,A)/\partial x = x^5 + A_1x^3 + A_2x^2 + A_3x + A_4 \quad (3)$$

In order to minimize the vulnerability of complex systems one can propose the methodology that includes the following stages: collection and analysis of initial information, definition of a function $U(x,A)$ on the basis of a set of experimental data, using the technology proposed in [5], mapping of set of source data to set of disaster management parameters with appropriate transformations, definition of indexes, which characterize the control parameters, based on a set of input data and appropriate mathematical models allowing us to determine the trajectory of the control parameters over time, calculation of the bifurcation surfaces, at the intersection of which the number or nature of the stationary states will be changed, the risk assessment of transition from one to another level of vulnerability in the degree of remoteness terms describing the current state of the bifurcation surfaces, which separate the different levels of security.

Shifts in the relation between A_i cause transitions from the norm to the pre-crisis or crisis states. Bifurcation values of these parameters can be calculated using the proposed mathematical methods. Achievement to these critical values sharply increases the probability of transition from one to another functional state. Thus, for a given system state, one can determine the range of parameters corresponding to the normal pre-crisis and crisis states.

The main advantage of the proposed methodology for modeling of appearance of instability in complex systems lies in the fact that it allows us to define the transformation of the vulnerability of a complex system as a function of dynamical variables.

Possible studies of instability in complex systems. The structuring of complex systems in a wide range of disciplines is carried out in [2-4, 6-11]. Algorithmic realization of interrelations in these systems allows justification of the form of function $U(x,A)$ based on the analysis of entry data. In tables 1 and 2, the classification of the control parameters used for the study of a number of systems in ecology, biology, psychology and sociology is shown.

The classification of the control parameters used for the study of global changes, economical, biological (health, neuro-immune-endocrine network (NIEN), cellular energy trigger, metabolic and hormonal regulation), epidemiological (the forecast of dissemination of infectious diseases, control of epidemic process and ranking of risks) and political systems by accident of "butterfly" type (3) is shown in Table 3.

Thus, the approach discussed above can be applied to various complex systems from ecology and economics to psychology and sociology, and can form the basis for the development of tools to extend the forecast horizon, develop a global strategy to prevent extreme and rare events, a lack of which is felt keenly today.

III. Study of the Effect of Global Change on Water Resources

Let X is the level of globalization, defined by levels of technological development, economical integration, intensity of personal contacts between people and political commitments, Y is the level of available water resources, Z is the level of instability related with the struggle for water resources. It was shown that the interrelation between these variables is described by Lorenz model of a metastable chaos [4]

$$dX/dt = \sigma(Y - X),$$
$$dY/dt = rX - Y - XZ, \quad (4)$$
$$dZ/dt = XY - bZ.$$

Here, σ, b, r characterize speeds of processes. Parameter r is the function of current supplies and demands on level of globalization and water resources. Fluctuations of parameter r cause significant transformations of the system dynamics. There are intervals of demand-supply ratio, which correspond to different modes of functionality. At the boundaries of such intervals, fluctuations can lead to catastrophic consequences associated with the transition from stability to instability (see figure).

TABLE I.

$\partial U/\partial x$	Control parameters	
	A_1	A_2
$x^3 + A_1x + A_2$ Type of disaster - "assembly"	Biological systems: calculation of the probability of appearance of pre-pathological and pathological states [2]	
	Determined by reserves of the biological system	Determined by the strength of the regulatory mechanisms
	Environmental systems: calculation of critical levels of pollution and ecosystem reserves [3, 4]	
	Determined by reserves of the environmental system	Determined by the level of environmental pollution
	Psychological systems: formation of the position	
	Depends on the emotional assessment of the situation in terms of its importance	Depends on a rational assessment of the situation in terms of probabilities of gains and losses

TABLE II.

$\partial U / \partial x$	Control parameters		
$x^4 + A_1 x^2 + A_2 x + A_3$	A_1	A_2	A_3
	Social systems: a study of the influence of physical, human and social capital to social security		
Type of disaster - "dovetail"	Depends on the index of sustainable development, which assesses the contribution of endogenous factors (economic, environmental and social) to the change in security	Determined by the index of globalization, which assesses the contribution of exogenous factors (shifts in the structure of relationships in a globalized world) to change the security	Determined by the index of social capital, assessing the degree of human corporations in society, allowing individuals to cooperate in the information society within a certain "radius of trust"

TABLE III.

Control parameters			
A_1	A_2	A_3	A_4
Global systems [6, 7]			
Depends on the ability to prevent / minimize the consequences of accidents	Depends on stimulant abuse, mental disadaptation, epidemics and suicides	Depends on the factors causing natural disasters due to climatic changes	Depends on factors that increase the risk of industrial accidents and human pressure
Economical systems [8]			
Depends on factors that characterize the labor market: the Gini index, unemployment, etc.	Depends on factors that characterize the financial market: loans, inflation, etc.	Depends on factors related to the securities market: business activity, etc.	Depends on factors related to market of goods and services: personal income, GDP, etc.
Biological systems [9, 10]			
Depends on the energy of cells and biologically active substances, hormones, NIEN	Depends on the balance of synthesis and energy expenditure, the adaptive capacity of cells	Depends on the psychic status of the body, forming a mental health	Depends on the genetic background (hereditary predisposition)
Epidemiological systems [11]			
Depends on the speed of propagation of pathogens and their resistance to drugs, etc.	Depends on the resistance to viruses, reserves and immune status	Depends on the quality of epidemiological services, availability of vaccines and equipment	Depends on environmental influences on the body's resistance to infections
Political systems			
Depends on the network of ideas - beliefs, proofs, definitions	Depends on the network rules - regulations, norms, ideals, values	Depends on the network of action - the ordering of status, hierarchy, community dialogue	Depends on the network of interests - opportunities, chances and access to resources

Chaotic oscillations ΔY obtained by the model (4) are shown in the figure *a*, where ΔY and time t are given in arbitrary units. As the results of observations show in [12], since 1860 there is a significant change in the oscillations ΔT, so that the time interval marked by the rectangle in the figure *b* is an example of the bifurcation zone in which there was a loss of stability of operation.

IV. CONCLUSION

This study involves the development of new modeling techniques to solve complex problems of adaptive control of behavior of systems in the unstable nonequilibrium medium with a strong dependence on initial conditions and strong information overloads. The proposed approach provides an approach to solving a number of fundamental issues related to the crisis of modern systems of control: to minimize the delay between the beginning of catastrophic changes and the moment of their detection; to transform the priorities from the response to the consequences (the tactics of "planning yesterday") to control the risk of events; to determine the optimal redistribution of resources and effective actions (controls), which minimize damage from natural and manmade disasters, as well as from terror actions; to rank the different types of threats, to identify weak links in a complex system, to assess its adaptability to a rapidly changing world.

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