

Genetic Algorithm for Multi Bandpass Problem and Library of Problems

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Abstract— Bandpass Problem (BP) is a telecommunication problem. This problem arises in considering the optimal packing of information flows on different wavelengths into groups to obtain the highest available cost reduction in design and operating the optical communication networks using wavelength division multiplexing technology. Given a rectangular matrix A of binary elements $\{0, 1\}$ and a positive integer B called the Bandpass Number, a set of B consecutive non-zero elements in any column is called a Bandpass. No two bandpasses in the same column can have common rows. The Bandpass problem consists of finding an optimal permutation of rows of the matrix, which produces the maximum total number of bandpasses having the same given bandpass number in all columns. In this paper, a new version of the BP named as the Multi Bandpass Problem (MBP) has been considered. An online library of the MBP including 45 test problems with known optimal solutions has been established. A meta-heuristic method, four Genetic Algorithm (GA) implementations have been first created for the MBP. These GAs have been tested on the MBP library problems and the results are listed.

Keywords— multi bandpass problem; telecommunication; library of problems; genetic algorithm

I. INTRODUCTION

A combinatorial optimization problem, called the Bandpass Problem, is often encountered in managing telecommunications networks and arises in considering the optimal packing of information flows on different wavelengths into groups to obtain the highest available cost reduction in design and operating the optical communication networks using wavelength division multiplexing technology.

Bandpass Problem was first introduced by Bell and Babayev in 2004 [1]. Babayev, Nuriyev, Bell, Kurt, Berberler and Gürsoy established an online Bandpass Problems library in order to compare the efficiency of the developed algorithms in 2007 [2]. Problem instances of this library are divided into two classes whose optimal solutions are known and unknown. Each of these classes has 45 samples. Babayev, Bell and Nuriyev proved that bandpass problem was in NP-hard in 2009 [3]. Then, Lin proved NP-Hardness of BP within a different way and developed polynomial algorithms for special cases in 2009 [6]. Next, Berberler and Gürsoy created a genetic algorithm approach for Bandpass Problems in 2010 [4]. After that, Li and Lin worked on solvability of the three column Bandpass problem in linear time in 2010 [7]. Lastly, Nuriyev, Kutucu and Kurt presented some new mathematical models and Ordermatic computer game in 2011 [8]. Also, the Multi Bandpass Problem

(MBP) has been introduced firstly by Nuriyev, Kutucu and Kurt [8].

In this paper, an algorithmic approach, which was first created for the MBP using genetic algorithm, was introduced. Problem instances used in computational experiments were taken from the online MBP library and the optimal solution values of these problems are known.

II. MULTI BANDPASS PROBLEM

Dense Wavelength Division Multiplexing (DWDM) technology is used in modern optical cable because of denser channel spacing. DWDM technology can carry information coded up to 160 different wavelengths [3].

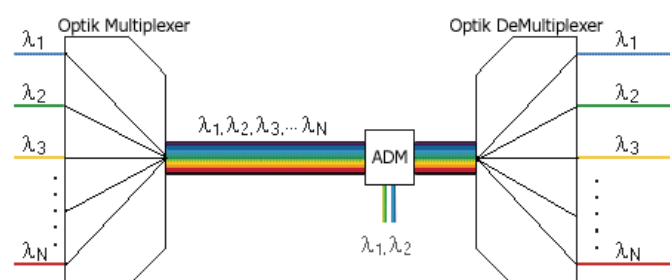


Figure 1. Multiplexing-demultiplexing of wavelengths and selecting wavelengths λ_1 and λ_2 by an ADM

Wavelengths ($\lambda_1, \lambda_2, \lambda_3 \dots$) are multiplexed to optic cable by using optic multiplexers and these are lastly left out optic cable by using optic demultiplexers (Figure 1). Information flows can leave the cable through special devices, add/drop multiplexers (ADM), installed on cable lines at destination points (Figure 1). In each ADM, special cards control each wavelength; the wavelengths may either pass through the ADM or dropped at their destination. [3, 1]

Although DWDM is very efficient in the long-distance communication, it is costly in the short distance because a lot of Add/Drops are required. So, many manufacturers research on how to employ cheaper DWDM systems on the Metropolitan Area Network (MAN).

The goal is to minimize the hardware and time costs while transmitting data from one point to other points. For this reason, it is needed to minimize using ADMs on DWDM systems.

Different wavelengths often must pass through the same ADM. If two wavelengths are adjacent on the cable then they are considered as consecutive. Hence these wavelengths may be arranged into a ring.

On an optical communication network, let a sending point has m information packages to be sent to n different destination points (Figure 2). If a package will be sent to a destination point, it is shown as 0, otherwise 1.

If we want to formulate this communication network as mathematically then the information packages refer to rows and the destination points refer to columns of a matrix. This situation is described by a Boolean matrix $A = \{a_{ij}\}$ of dimension $m \times n$. If information package $i = 1, \dots, m$ is destined for point $j = 1, \dots, n$, then $a_{ij} = 1$, otherwise $a_{ij} = 0$. Figure 2 shows that Package 1 will be sent to the Destination 2 but not sent to the Destination 1 etc. In Figure 2, if the bandpass number is five for destination 1 then wavelengths $\lambda_4, \lambda_5, \lambda_6, \lambda_7$ and λ_8 can be grouped as a bandpass because these five wavelengths are consecutive. So, destination 1 is able to optimize using just one card instead of the five.

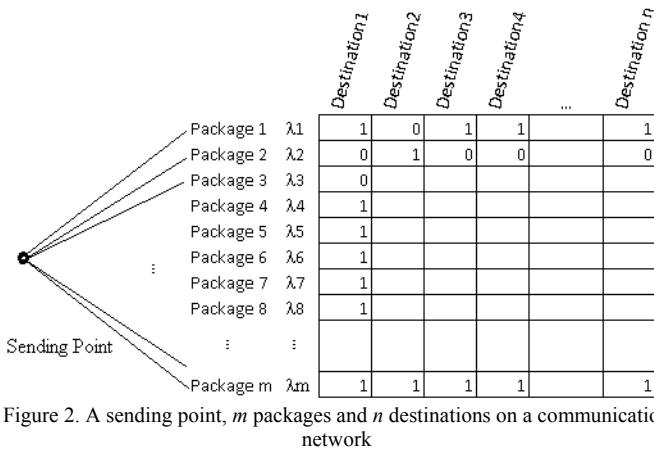


Figure 2. A sending point, m packages and n destinations on a communication network

r\c	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	0	1	0	0	0
8	0	1	1	1	1
9	0	1	1	1	1
10	0	1	1	1	1
11	1	1	1	1	1

B	6	5	4	8	7
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Figure 3. One bandpass case for $B_1=6$ on column 1.

For example, consider a 0-1 matrix A of dimension 11 by 5 as in Figure 3, and the bandpass vector $B_j = (6, 5, 4, 8, 7)$. Column 1 has a bandpass in Figure 3 and only this case has to be chosen. Column 2 has two bandpasses. There are also three

possible cases (a, b and c cases in Figure 4) and only one of them can be selected. Lastly, column 3 has two bandpasses and there is one case to be selected in Figure 5. There are no bandpasses on columns 4 and 5 because there are no $B_4=8$ for column 4 and $B_5=7$ for column 5 consecutive non-zero elements. When the matrix is used as given, only five bandpasses can be obtained.

r\c	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	0	1	0	0	0
8	0	1	1	1	1
9	0	1	1	1	1
10	0	1	1	1	1
11	1	1	1	1	1

B	6	5	4	8	7
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(a)

r\c	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	0	1	0	0	0
8	0	1	1	1	1
9	0	1	1	1	1
10	0	1	1	1	1
11	1	1	1	1	1

B	6	5	4	8	7
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(b)

r\c	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	0	1	0	0	0
8	0	1	1	1	1
9	0	1	1	1	1
10	0	1	1	1	1
11	1	1	1	1	1

B	6	5	4	8	7
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(c)

Figure 4. Three different bandpass cases (a, b and c) for $B_2=5$ on column 2.

BP can be solved as finding an optimal permutation of rows of the matrix. The optimum permutation can be found by enumerating all permutations and the permutations are bounded with 11! for this example. It is easy to say that the optimum solution of the matrix A has seven bandpasses when the optimum permutation of rows is sorted as 1-2-3-4-5-6-11-8-9-10-7 (Figure 6).

The MBP is different from the BP. BP has a fixed Bandpass number for all columns. However, in the MBP, each column has a specific bandpass number which can be different from others. The MBP is a problem of finding an optimal packing of information flows on different wavelengths having specific sized bandpasses for the destinations in the telecommunication networks and it is a combinatorial optimization problem.

r\c	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	0	1	0	0	0
8	0	1	1	1	1
9	0	1	1	1	1
10	0	1	1	1	1
11	1	1	1	1	1

B	6	5	4	8	7
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Figure 5. One bandpass case for $B_3=4$ on column 3.

r\c	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	0	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
11	1	1	1	1	1
8	0	1	1	1	1
9	0	1	1	1	1
10	0	1	1	1	1
7	0	1	0	0	0

B	6	5	4	8	7
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Figure 6. The optimum solution (permutation) of matrix A

In the mathematical model of the MBP, consider a Boolean matrix $A_{m \times n}$ and an integer bandpass vector B_n . Let x_{ik} and y_{ij} be decision variables as follows:

$$x_{ik} = \begin{cases} 1, & \text{if row } i \text{ is relocated to position } k; \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if row } i \text{ is the first row of a bandpass in column } j; \\ 0, & \text{otherwise.} \end{cases}$$

The model can be formulated as follows [8]:

$$\max \sum_{j=1}^n \sum_{k_j=1}^{m-B_j+1} y_{k_j j} \quad (1)$$

subject to

$$\sum_{k=1}^m x_{ik} = 1, i = 1, \dots, m \quad (2)$$

$$\sum_{i=1}^m x_{ik} = 1, k = 1, \dots, m \quad (3)$$

$$\sum_{i=k_j}^{k_j+B_j-1} y_{ij} \leq 1, j = 1, \dots, n, \quad (4)$$

$$k_j = 1, \dots, m - B_j + 1$$

$$B_j \cdot y_{k_j j} \leq \sum_{i=k_j}^{k_j+B_j-1} \sum_{r=1}^m a_{rj} x_{ri}, \quad (5)$$

$$j = 1, \dots, n, k_j = 1, \dots, m - B_j + 1$$

$$x_{ik}, y_{k_j j} \in \{0, 1\}, i = 1, \dots, m, j = 1, \dots, n, \quad (6)$$

$$k = 1, \dots, m,$$

$$k_j = 1, \dots, m - B_j + 1$$

Under the constraints (2-6), the goal is to maximize the bandpasses of the matrix A (1). This model is named as the Boolean Integer Mathematical Model of the Multi Bandpass Problem [8].

III. GENETIC ALGORITHMS, COMPUTATIONAL EXPERIMENTS AND LIBRARY OF PROBLEMS

The solution of the MBP is a permutation of the input problem. The MBP is structurally similar to the Traveling Salesman Problem (TSP) but adaptation of the MBP to Genetic Algorithm (GA) is different from the TSP. For the solution, m rows must be replaced so that new $m \times n$ dimensional matrix has the largest total number of bandpass. GA population makes it difficult to obtain efficient solutions because of only randomly generated permutations of rows. The fitting value of a permutation (or chromosome) is the number of bandpasses of the matrix [5].

To solve the MBP with GA, two crossover and two mutation operators were used. In the beginning of the GA, initial population size (ps), crossover rate (cr), mutation rate (mr), crossover number (cn) and mutation number (mn) are determined as below:

m : row number,
 n : column number,
 B_j : bandpass width of column j ,
 d : density of the matrix.

$$ps = 10 \cdot m$$

$$cr = 0.9$$

$$mr = 1 - cr$$

$$cn = cr \cdot ps$$

$$mn = mr \cdot ps$$

Initial set of population is ordered non-increasingly by the fitting values. The GAs are ended by a stopping criteria that repeating best fitting value obtained during 100 generations.

It is used two main crossover methods. The crossovers are different from each other by selection. The first method selects two parents using roulette wheel selection and this o is repeated cn times. The next method crosses first (respectively the best) cn parents with randomly selected parents.

There are two operators for the mutation process. The first is the two-point mutation. When a chromosome selected according to desired criteria, randomly selected two genes are exchanged. To increase the effectiveness of this method, the new chromosome is compared to parents. The chromosome, whose fitness value is greater in the population, continues to live while the others are destroyed. In the literature this method is referred to as 2-opt. Other method mutates first (best) *mn* chromosomes and uses 2-opt.

As a combinatorial optimization problem having important applications in telecommunication sector, a library of the Multi Bandpass Problems is created. The MBP library includes 45 problem instances with known optimal solutions to challenge for researchers. [9]

45 problem instances from the MBP library [9] are listed with row number (*m*), column number (*n*) and bandpass range (*b*) in Table I, and each instance includes the optimal value (Opt), four metaheuristic solutions (g1, g2, g3 and g4) obtained from the GAs and related efficiencies (ef.).

TABLE I. Solutions and efficiencies of genetic algorithms on library problems

Problem	Opt	g1	ef.	g2	ef.	g3	ef.	g4	ef.
OS-P1(m64,n8,b5-8)	18	7	39 %	8	44 %	8	44 %	8	44 %
OS-P2(m64,n8,b5-16)	17	1	65 %	1	59 %	1	59 %	1	65 %
OS-P3(m64,n8,b8-16)	11	6	55 %	5	45 %	5	45 %	5	45 %
OS-P4(m64,n8,b8-25)	15	9	60 %	1	67 %	9	60 %	9	60 %
OS-P5(m64,n8,b16-25)	14	1	71 %	0	64 %	8	57 %	9	64 %
OS-P6(m64,n8,b16-32)	12	5	42 %	4	33 %	4	33 %	5	42 %
OS-P7(m64,n12,b5-8)	25	8	32 %	9	36 %	9	36 %	9	36 %
OS-P8(m64,n12,b5-16)	43	2	56 %	2	58 %	2	58 %	2	56 %
OS-P9(m64,n12,b8-16)	22	8	36 %	9	41 %	0	45 %	7	32 %
OS-P10(m64,n12,b8-25)	32	2	63 %	1	50 %	1	53 %	1	47 %
OS-P11(m64,n12,b16-25)	13	5	38 %	5	38 %	5	38 %	6	46 %
OS-P12(m64,n12,b16-32)	16	8	50 %	7	44 %	7	44 %	7	44 %
OS-P13(m64,n16,b5-8)	31	1	39 %	1	42 %	1	39 %	9	29 %
OS-P14(m64,n16,b5-16)	49	2	43 %	2	45 %	2	47 %	2	41 %
OS-P15(m64,n16,b8-16)	31	1	52 %	1	45 %	1	42 %	1	35 %
OS-P16(m64,n16,b8-25)	45	2	58 %	2	58 %	2	58 %	2	64 %
OS-P17(m64,n16,b16-25)	28	1	54 %	1	54 %	1	57 %	1	54 %
OS-P18(m64,n16,b16-32)	26	1	65 %	1	69 %	1	69 %	1	65 %
OS-P19(m96,n8,b8-25)	15	3	20 %	3	20 %	4	27 %	4	27 %

Problem	Opt	g1	ef.	g2	ef.	g3	ef.	g4	ef.
OS-P20(m96,n8,b8-32)	25	1	40 %	9	36 %	1	40 %	1	40 %
OS-P21(m96,n8,b16-25)	19	5	26 %	6	32 %	4	21 %	5	26 %
OS-P22(m96,n8,b16-32)	20	8	40 %	8	40 %	6	30 %	7	35 %
OS-P23(m96,n16,b8-25)	46	8	17 %	9	20 %	9	20 %	0	22 %
OS-P24(m96,n16,b8-32)	46	1	35 %	1	37 %	1	35 %	1	33 %
OS-P25(m96,n16,b16-25)	37	6	22 %	7	19 %	6	16 %	8	22 %
OS-P26(m96,n16,b16-32)	40	2	50 %	1	48 %	1	35 %	1	43 %
OS-P27(m96,n25,b8-25)	59	0	20 %	0	17 %	0	17 %	1	19 %
OS-P28(m96,n25,b8-32)	66	2	30 %	1	24 %	2	35 %	2	30 %
OS-P29(m96,n25,b16-25)	56	9	16 %	2	21 %	9	16 %	8	14 %
OS-P30(m96,n25,b16-32)	61	2	36 %	2	38 %	2	34 %	2	34 %
OS-P31(m64,n8,b16-25)	18	1	78 %	1	72 %	1	78 %	1	72 %
OS-P32(m64,n12,b8-32)	29	2	69 %	1	66 %	2	69 %	1	66 %
OS-P33(m64,n16,b5-8)	89	6	70 %	5	58 %	5	61 %	5	61 %
OS-P34(m64,n25,b5-16)	96	2	65 %	2	57 %	4	63 %	4	50 %
OS-P35(m64,n25,b8-16)	88	6	65 %	5	57 %	6	63 %	4	50 %
OS-P36(m64,n25,b16-25)	54	2	74 %	3	69 %	4	74 %	3	65 %
OS-P37(m96,n8,b16-25)	29	0	62 %	1	59 %	1	62 %	1	59 %
OS-P38(m96,n25,b16-25)	89	8	63 %	5	62 %	5	62 %	5	56 %
OS-P39(m96,n8,b5-16)	58	5	63 %	5	62 %	5	62 %	5	56 %
OS-P40(m96,n8,b8-25)	35	3	64 %	3	59 %	3	64 %	3	59 %
OS-P41(m96,n16,b16-32)	48	7	74 %	2	74 %	2	74 %	2	71 %
OS-P42(m96,n16,b5-25)	75	6	54 %	6	54 %	4	50 %	5	52 %
OS-P43(m96,n16,b25-32)	26	5	72 %	5	68 %	4	65 %	4	60 %
OS-P44(m96,n25,b8-25)	105	4	19 %	1	19 %	9	27 %	5	19 %
OS-P45(m96,n25,b25-32)	53	5	54 %	5	50 %	5	53 %	5	54 %

IV. CONCLUSION

In this paper, a library to challenge for researchers for the MBP was firstly created and 45 problem instances with known optimal solutions were generated to compare efficiency. Then, four GA implementations including different crossover and mutation operators were the first time created and they were run on the 45 library problems. Computational experiments of the GAs were listed and the efficiencies were compared.

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