# NUMERICAL STUDY OF VORTEX PATTERN IN TWO-BAND SUPERCONDUCTORS

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In the present study, the vortices nucleation (singular solution) of vortex in external magnetic field in the framework of a two-band model two-band GL equations presented. Firstly we will drive time-dependent GL equations for two-band superconductors. Secondly we apply these equations for numerical modeling for vortex nucleation in the case thin superconducting film of two-band superconductor *MgB2* with perpendicular external magnetic field. We could use the modified forward Euler method for numerical experiments. Finally, a conclusion remarks will be made.

Time-dependent equations in two-band Ginzburg-Landau theory can be obtained from expression for free energy functional [1-4] in analogical way to [5]:

$$\Gamma_1(\frac{\partial}{\partial t}+i\frac{2e}{\hbar}\phi)\Psi_1=-\frac{\delta F}{\delta\Psi_1^*},$$

$$\Gamma_2(\frac{\partial}{\partial t}+i\frac{2e}{\hbar}\phi)\Psi_2=-\frac{\delta F}{\delta\Psi_2^*}, \tag{1}$$

$$\sigma_n(\frac{\partial\vec{A}}{\partial t}+\nabla\phi)=-\frac{1}{2}\frac{\delta F}{\delta\vec{A}}$$

Here we use notations similar to [5]. In Eqs. (6) $\varphi$ means electrical scalar potential , $\Gamma_{1,2}$ - relaxation time of order parameters, $\sigma_n$-conductivity of sample in two-band case, F denotes free energy functional of two-band superconductors [1-4] . Choosing corresponding gauge invariance we can eliminate scalar potential from system of equations (1) [5]. Under such calibration and magnetic field in form, $\vec{H}=(0,0,H)$ without any restriction of generality, time-dependent equations in two-band Ginzburg-Landau theory can be written as

$$\Gamma_1\frac{\partial\Psi_1}{\partial t}=-\frac{\hbar^2}{4m_1}(\frac{d^2}{dx^2}-\frac{x^2}{l_s^4})\Psi_1+\alpha_1(T)\Psi_1+\varepsilon\Psi_2+\varepsilon_1(\frac{d^2}{dx^2}-\frac{x^2}{l_s^4})\Psi_2+\beta_1\Psi_1^3=0, \tag{2a}$$

$$\Gamma_2\frac{\partial\Psi_2}{\partial t}=-\frac{\hbar^2}{4m_2}(\frac{d^2}{dx^2}-\frac{x^2}{l_s^4})\Psi_2+\alpha_2(T)\Psi_2+\varepsilon\Psi_1+\varepsilon_1(\frac{d^2}{dx^2}-\frac{x^2}{l_s^4})\Psi_1+\beta_2\Psi_2^3=0, \tag{2b}$$

$$\sigma_n(\frac{\partial\vec{A}}{\partial t}-\nabla\phi)=-rot\vec{A}+\frac{2\pi}{\Phi_0}\{\ \frac{\hbar^2}{4m_1}n_1(T)(\frac{d\varphi_1}{dr}-\frac{2\pi A}{\Phi_0})+\varepsilon_1(n_1(T)n_2(T))^{0.5}\cos(\varphi_1-\varphi_2)+ \frac{\hbar^2}{4m_2}n_2(T)(\frac{d\varphi_2}{dr}-\frac{2\pi A}{\Phi_0})\ \}, \tag{2c}$$

*where mi* are the masses of electrons belonging to different bands ($i$ = 1, 2); α*i* = γ*i*($T$ – $Tci$) are the quantities linearly dependent on the temperature; β and γ*i* are constant coefficients; ε and ε1 describe the interaction between the band order parameters and their gradients, respectively; $H$ is the external magnetic field; and *Φ0* is the magnetic flux quantum. In Eqs. (2,3.4), the order parameters are assumed to be slowly varying in space.

In Eqs. (2-4) was introduced also $\phi_{1,2}(\vec{r})$ phase of order parameters $\Psi_{1,2}(\vec{r}) = |\Psi_{1,2}|\exp(i\phi_{1,2})$, $n_{1,2}(T) = 2|\Psi_{1,2}|^2$ -density of superconducting electrons in different bands, expressions for whichs are presented in [16–19] with so-called natural boundary conditions

$$\left\{\frac{1}{4m_1}(\nabla - \frac{2\pi i\vec{A}}{\Phi_0})\Psi_1 + \varepsilon_1(\nabla - \frac{2\pi i\vec{A}}{\Phi_0})\Psi_2 \right\}\vec{n} = 0 , \tag{5}$$

$$\left\{\frac{1}{4m_2}(\nabla - \frac{2\pi i\vec{A}}{\Phi_0})\Psi_2 + \varepsilon_1(\nabla - \frac{2\pi i\vec{A}}{\Phi_0})\Psi_1 \right\}\vec{n} = 0 , \tag{6}$$

$$(\vec{n} \times \vec{A}) \times \vec{n} = \vec{H}_0 \times \vec{n} \tag{7}$$

First two conditions correspond to absence of supercurrent through boundary of two-band superconductor, third conditions correspond to the contiunity of normal component of magnetic field to the boundary superconductor-vacuum.

We consider a finite homogeneous superconducting film of uniform thickness, subject to a constant magnetic field. We also consider that the superconductor is rectangular in shape. In this case our two-band GL model becomes two-dimensional. The order parameters $\Psi_1$ and $\Psi_2$ varies in the plane of the film, and vector potential **A** has only two nonzero components, which lie in the plane of the film. Therefore, we identify the compuational domain of the superconductor with a rectangular region $\Omega \in \mathbb{R}^2$, denoting the Cartesian coordinates by $x$ and $y$, and the $x$- and $y$- components of the vector potential by $A(x,y)$ and $B(x,y)$, recpectively. Before modeling we use so-called bond variables [7] for the discretization of time-dependent two-band G-L equations

$$W(x,y) = \exp(i\kappa\int^{x} A(\zeta, y)d\zeta) , \tag{8}$$

$$V(x,y) = \exp(i\kappa\int^{y} B(x,\eta)d\eta) \tag{9}$$

Such variables make obtained discretized equations gauge-invariant. For spatially discetization we use forward Euler method [7]. In this method we begin with partitioning the computational domain $\Omega = \left[0, N_{xp}\right] \times \left[0, N_{yp}\right]$ into two subdomains, denoted by $\Omega_{2n}$ and $\Omega_{2n+1}$ such that

$$\Omega_{2n} = \Omega \Big|_{i+j=2n} \text{ and } \Omega_{2n+1} = \Omega \Big|_{i+j=2n+1} \tag{10}$$

for $i = 0,.....;N_{xp}, j = 0,.....;N_{yp}$, where $N_{xp} = N_x + 1$, $N_{yp} = N_y + 1$. In calculations we could use two different approach. The first approach (zero-field –cooled) is assume that sample that has is initially in a perfect superconducting state is cooled to a temperature below the critical $T_c$ in the absence of applied magnetic field, and then a magnetic field of an appropriate strength is suddenly turned out. The second approach (field-cooled) is to assume that a sample that is cooled to a temperature at or above the critical temperature is in a normal state under magnetic field of appropriate strength, and then the temperature is suddenly decreased below the critical temperature.

For numerical calculations in two-band Ginzburg-Landau theory we assume that the size of superconducting film is $40\lambda \times 40\lambda$, where $\lambda$ London penetration depth of external magnetic field on superconductor [1-4]. Expressions for $\Psi_{(1,2)0}$, and for thermodynamic magnetic field $H_c$ are also presented in [1–4]. The calculations were performed for the following values of parameters: $T_c$ = 40 K; $T_{c1}$ =20.0 K; $T_{c2}$ = 10 K, $\frac{\varepsilon^2}{\gamma_1\gamma_2T_c^2} = 3/8$ ; $\eta = \frac{T_c m_2 \varepsilon_1 \gamma_2}{\hbar^2 \varepsilon} = -0.016$. This parameters was used for the calculation another physical properties of two-band superconductor *MgB2* [1-4].

For solving of corresponding discretized GL equations we will use method of adaptive grid [7]. We assume that the sample, which is initially in a perfect superconducting state, is cooled through *Tc* in the absence of applied magnetic field, and then a magnetic field of an appropriate strenght is suddenly turned out. Mathematically it means that, the initial state is achieved by letting

$$\left| \Psi'_{1,2}(\vec{x}) \right| = 1,\ A_0(\vec{x}) = 0 \text{ for all } \vec{x} \in \Omega. \tag{11}$$

We present a contour plot of superconducting electrons. Ginzburg-Landau parameter for sample is the $\kappa = 5$. We can observe a partial hexagonal pattern, yet we do not observe the physically exact hexagonal pattern, as expected of homogeneous samples with uniform thickness.

Secondly we simulate the field cooled case. In $(x_0, y_0)$ a temperature at or above the critical temperature, is in a normal state under a magnetic field of appropriate strenght, and then the temperature is suddenly reduced to below *Tc*. In matematical denotes, the initial states is achieved by letting

$$A_0(x,y) = (0, xH, 0),\ \left| \Psi'_{1,2}(x,y) \right| = \begin{cases} 0, if\,(x,y) \neq (x_0, y_0) \\ c_{1,2}, f\,(x,y) = (x_0, y_0) \end{cases}, \tag{12}$$

where $c_{1,2}$ is a small constant representing the magnitude of the seed, and $(x_0, y_0)$ is the location of a seed in the sample. We can conclude that, the result vortex pattern depends upon where and how many seeds are placed into the sample. Existence of Meissner state is shown by numerical calcutions using both (zero-field-cooled and field cooled) approachs. It means that at fixed Ginzburg-Landau parameter $\kappa$ and external magnetic field $H < H_{c1}$ no nucleation of vortexes of external magnetic field.

As shown in [8] structure of magnetic field in section of vortex in two-band superconductor differs from single-band superconductor. Nonsymmetric angular magnetic field distriburion in vortex change their interaction force between them and total energy of superconductor with such vortexes differs from single band one. In high density vortex pattern effects of influence of nonsymmetric angular dependence becomes crusial. Detail analysis of influence of asymmetric character of sectional magnetic field distribution on the parameters of hexagonal vortex pattern is the object of future investigations.

Thus, in this study we obtain time-dependent Ginzburg-Landau equations taking into account two-band character of the superconducting state, which was originally developed by Schmid for single band superconductors. Furthermore, we perform numerical modeling of vortex nucleation in external magnetic field in two-band superconducting films *MgB2* using two-band Ginzburg-Landau theory. It was shown that the vortex configuration in the mixed state depends upon initial state of the sample and that the system does not seem to yield hexagonal pattern for finite size homogeneous samples of uniform thickness with the natural boundary conditions. On the other hand, the time-dependent two-band GL equations leads to the expected hexagonal pattern, i.e. global minimizer of the energy functional.

## References

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