# EVENTOLOGY SIGHT AT ENTROPY

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Eventology – the new direction which based on probabilities theory. The reasonable subject enters into scientific research as strict mathematical concept – eventological distribution (E-distribution) – not only in humanitarian and social and economic area, but also in the field of natural-science [2]. Eventology studies the person through its own E-distribution reflected in corresponding subsets of its own events, analyzing dependences and independences of the person from itself and from a society.

The term entropy is widely applied in various areas of knowledge. Probabilistic treatment of statistical entropy (or entropy of Boltzmann) and information entropy (entropy of Shannon) are projected on entropy in eventology.

Let ($\Omega$, $\Phi$, **P**) is the probability space, $\mathfrak{X} \subseteq \Phi$ - finite set of the events chosen from algebra of events $\Phi$ of this space. Each $N$-set of events is characterised by a set from $2^N$ probabilities. The given set is called E-distribution of set of events of $\mathfrak{X}$ of the power $N$, i.e. $N = |\mathfrak{X}|$. E-distribution of set of events $\mathfrak{X}$ can be defined by one of sets of probabilities of events-terraces among which it is possible to allocate three most used sets at $X \subseteq \mathfrak{X}$:

$$p(X) = \mathbf{P}(\mathrm{ter}(X)) = \mathbf{P}\left(\bigcap_{x \in X} x \bigcap_{x \in X^{\mathrm{c}}} x^{\mathrm{c}}\right),\ p_X = \mathbf{P}(\mathrm{ter}_X) = \mathbf{P}\left(\bigcap_{x \in X} x\right),\ p^X = \mathbf{P}(\mathrm{ter}^X) = \mathbf{P}\left(\bigcap_{x \in X^{\mathrm{c}}} x^{\mathrm{c}}\right).$$

Sets of probabilities we will name E-distributions of *I* th, *II* th or *III* th sort, and probability from sets – probabilities of *I* th, *II* th or *III* th sort accordingly.

Entropy of E-distribution $\{p(X), X \subseteq \mathfrak{X}\}$ of set of events $\mathfrak{X}$ is defined under the classical formula the same as entropy of probability distribution, or entropy of a random variable with finite number of values is defined in probability theory: $H(\mathfrak{X}) = -\sum_{X \subseteq \mathfrak{X}} p(X) \ln p(X)$, also it can be interpreted as a measure of uncertainty of E-distribution of set of events [1], [2], [3].

In work [3] the concept of wide dependence of events, as families of structures of probabilistic dependences of the events defined by characteristic parities and restrictions with which the probabilistic measure is obliged to satisfy is entered. As the convenient tool of the analysis of structures eventological dependences serves a multicovariance of events [5], as a measure of a multiplicate deviation of set of events from an independent situation. It is irreplaceable at studying of classes of Gibbsean and anti-Gibbsean E-distributions which connect probability with entropy, with an information and with a value of events. The device of multicovariances is widely applied in researches of these E-distributions or to it similar, and also in modelling by means of such E-distributions humanitarian and socio-economic systems: «perception and activity» in psychology, «supply and demand» in Economics. Multicovariances use in the analysis and the decision of variation problems eventological theories of information in which search for equilibrium E-distributions of sets of the events, providing an extremum of entropy or relative entropy at any restrictions on those or other subsets of events.

In the book [2, p. 113, 160] definitions multicovariances (*I* th, *II* th or *III* th sorts) any subset of events are given $X \subseteq \mathfrak{X}$. For example, multicovariance of *I* th sorts can be defined formulas of the reference of Mőbius [5]:

$$\tau(X) = \prod_{Y \subseteq X} (p(Y))^{(-1)^{|X-Y|}}, \quad p(X) = \prod_{Y \subseteq X} \tau(Y).$$

Feature of multicovariancical E-distributions are areas of possible values of multicovariances: $\tau(X)$, $\tau_X$, $\tau^X$ $\in [0, \infty]$ which wider in comparison with the individual intervals intended for values of probabilities: $p(X)$, $p_X$, $p^X$ $\in [0, 1]$. Advantage of multicovariances is value of unit which appears in the field of possible values multicovariances ($\tau(X) = 1$, $\tau_X = 1$, $\tau^X = 1$). That means reduction of set of parametres, sufficient for analysis of characteristic of the E-distributions of set of events X. E-distributions maximising entropy at known restrictions, are defined in O.Yu. Vorobyev's work [3]. Their role is played by is wide-multiplicate lower approximations of corresponding power.

Multiplicative-truncated approximation of the order $N_0$ of E-distribution $\{p(X), X \subseteq \mathfrak{X}\}$ it is defined as E-distribution $\{\hat{p}^{[N_0]}, X \subseteq \mathfrak{X}\}$, in which $\hat{p}^{[N_0]}(X) = \frac{1}{Z_{N_0}} \prod_{m=0}^{N_0} p^{[m]}(X)$, where $Z_{N_0} = \sum_{X \subseteq \mathfrak{X}} \prod_{m=0}^{N_0} p^{[m]}(X)$ - the multiplier providing global normalizing relation of multiplicative-truncated approximation $\hat{p}^{[N_0]}$, $[N_0]$ any order $N_0 = 0, 1, ..., |\mathfrak{X}|$, and $p^{[m]}(X) = \prod_{Y_m \subseteq X} \tau(Y_m)$, $X \subseteq \mathfrak{X}$, - multiplicative-truncated projection of the order $m$ of E-distribution $p(X)$, $\{\tau(Y), Y \subseteq \mathfrak{X}\}$ - multicovariances of E-distribution $p(X)$.

The set of events X is called as full if in its E-distribution of *I* th sort all $2^{|\mathfrak{X}|}$ probabilities do not address in zero: $p(X) > 0$, $X \subseteq \mathfrak{X}$. E-distributions of full sets of events are called as full E-distributions. Let's notice that each $m$- multiplicative projection $p^{[m]}(X)$ any full E-distribution $p(X)$ is defined on $2^{|\mathfrak{X}|}$ only by the values $p^{[m]}(Y_m) = \tau(Y_m)$ on $m$- subsets $Y_m \subseteq \mathfrak{X}$, which quantity is equal $C_{|\mathfrak{X}|}^m$. For multiplicative-truncated approximation $\hat{p}^{[N_0]}$, as E-distribution, entropy which is calculated under the formula from [3] is defined::

$$H_{\hat{p}^{[N_0]}} = -\sum_{X \subseteq \mathfrak{X}} \hat{p}^{[N_0]}(X) \ln \hat{p}^{[N_0]}(X). \tag{1}$$

Entropy of E-distribution $\{p(X), X \subseteq \mathfrak{X}\}$ concerning its multiplicative-truncated approximation $\hat{p}^{[N_0]}$ is defined by the formula:

$$H_{p/\hat{p}^{[N_0]}} = \sum_{X \subseteq \mathfrak{X}} p(X) \ln \frac{p(X)}{\hat{p}^{[N_0]}(X)}, \tag{2}$$

as a measure of rapprochement E-distribution $p(X)$ sets of events X in relation to the multiplicative-truncated approximations [3]. Using such important circumstance proved in work [1] that at fixation of probabilities of *II* th sort, the maximum of entropy of the E-distributions of *I* th sort is reached on E-distributions of *I* th sort [1], [2], in [7] the following is proved:

**Theorem (about entropy of E-distribution concerning its multiplicative-truncated approximation).** *For any full E-distribution p, II th sort having fixed probability on subsets of events to power [N₀] inclusive, relative entropy of E-distribution p in relation to its multiplicative-truncated approximation* $\hat{p}^{[N_0]}$ *of any order* $N_0 = 0, 1, ..., |\mathfrak{X}|$ *is equal to a difference between entropy of approximation and entropy of the distribution:*

$$H_{p/\hat{p}^{[N_0]}} = -H_p + H_{\hat{p}^{[N_0]}}.$$

Let's illustrate the theorem on a simple example. Let's consider the probability space ($\Omega$, $\Phi$, **P**) and any full 2-plet of the events $\mathfrak{X} = \{x, y\} \subseteq \Phi$, chosen from algebra of events $\Phi$ of this space. Any full 2-plet of the events X has full E-distribution of *I* th sort of a kind $\{p(\varnothing), p(x),$

$p(y), p(x, y)\}$ in which by definition: $p(X) > 0, X \subseteq \mathfrak{X}$ и $\sum_{X \subseteq \mathfrak{X}} p(X) = 1$. E-distribution of *II* th sort for this 2-plet $\{p_\varnothing, p_x, p_y, p_{xy}\}$, consisting of probabilities of *II* th sort, is connected with E-distribution of *I* th sort under formulas of the reference of Mőbius

$$p_X = \sum_{X \subseteq Y} p(Y), \quad p(X) = \sum_{X \subseteq Y} (-1)^{|Y|-|X|} p_Y.$$

Let's write formulas for the given 2-plet:

$$p_\varnothing = p(\varnothing) + p(x) + p(y) + p(x, y) = 1; \ p_x = p(x) + p(x, y) = \mathrm{P}(x);$$
$$p_y = p(y) + p(x, y) = \mathrm{P}(y); \ p_{xy} = p(x, y) = \mathrm{P}(x \cap y).$$

E-approximation of zero-power for E-distribution $\{p(X), X \subseteq \mathfrak{X}\}$ is equal $\hat{p}^{[0]}(X) = \dfrac{p^{[0]}(X)}{\sum_{X \subseteq \mathfrak{X}} p^{[0]}(X)} = \dfrac{p(\varnothing)}{p(\varnothing) \cdot 2^{|\mathfrak{X}|}} = \dfrac{1}{2^{|\mathfrak{X}|}}$, i.e. coincides with equiprobable on $2^{|\mathfrak{X}|}$ E-distribution. For 2-plet of the events $\mathfrak{X} = \{x, y\}$ 0- approximation is equiprobable E-distribution of *I* th sort: $\left\{\hat{p}^{[0]}(\varnothing), \hat{p}^{[0]}(x), \hat{p}^{[0]}(y), \hat{p}^{[0]}(x, y)\right\} = \{1/4, 1/4, 1/4, 1/4\}$ and of *II* th sort: $\left\{\hat{p}^{[0]}_\varnothing, \hat{p}^{[0]}_x, \hat{p}^{[0]}_y, \hat{p}^{[0]}_{xy}\right\} = \{1, 1/2, 1/2, 1/4\}$. From (1) entropy of equiprobable E-distribution of 2-plet of the events $\mathfrak{X} = \{x, y\}$ is equal $H_{\hat{p}^{[0]}} = -\sum_{X \subseteq \mathfrak{X}} \hat{p}^{[0]}(X) \ln \hat{p}^{[0]}(X) = -\sum_{X \subseteq \mathfrak{X}} 1/4 \cdot \ln 1/4 = \ln 4.$

E-approximation of the first power for 2-plet of the events $\mathfrak{X} = \{x, y\}$ is equal

$$\hat{p}^{[1]}(X) = \frac{p^{[0]}(X) \cdot p^{[1]}(X)}{\sum_{X \subseteq \mathfrak{X}} p^{[0]}(X) \cdot p^{[1]}(X)} = \frac{p^{[1]}(X)}{\sum_{X \subseteq \mathfrak{X}} p^{[1]}(X)} = \frac{p^{[1]}(X)}{1 + \tau(x) + \tau(y) + \tau(x) \cdot \tau(y)}, \quad \sum_{X \subseteq \mathfrak{X}} \hat{p}^{[1]}(X) = 1.$$

Let's designate a denominator through $Z_1 = 1 + \tau(x) + \tau(y) + \tau(x) \cdot \tau(y)$. Then $\left\{\hat{p}^{[1]}(\varnothing), \hat{p}^{[1]}(x), \hat{p}^{[1]}(y), \hat{p}^{[1]}(x, y)\right\} = \{1/Z_1, \tau(x)/Z_1, \tau(y)/Z_1, \tau(x) \cdot \tau(y)/Z_1\}$. From (1) follows that entropy of E-approximation of the first power for 2-plet of the events $\mathfrak{X} = \{x, y\}$ is equal

$$H_{\hat{p}^{[1]}} = -\sum_{X \subseteq \mathfrak{X}} \hat{p}^{[1]}(X) \ln \hat{p}^{[1]}(X) = \ln Z_1 - \ln \tau(x) \cdot \left[\hat{p}^{[1]}(x) + \hat{p}^{[1]}(x, y)\right] -$$
$$- \ln \tau(y) \cdot \left[\hat{p}^{[1]}(y) + \hat{p}^{[1]}(x, y)\right] = \ln Z_1 - \ln \tau(x) \cdot \hat{p}^{[1]}_x - \ln \tau(y) \cdot \hat{p}^{[1]}_y.$$

E-approximation of the first power is always the E-distribution independent in aggregate defined only by probabilities of events which for $\mathfrak{X} = \{x, y\}$ looks like *I* th sort:

$$\left\{\hat{p}^{[1]}(\varnothing), \hat{p}^{[1]}(x), \hat{p}^{[1]}(y), \hat{p}^{[1]}(x, y)\right\} = \{(1 - p_x)(1 - p_y), \ p_x(1 - p_y), \ (1 - p_x)p_y, \ p_x p_y\}$$

and of *II* th sort: $\left\{\hat{p}^{[1]}_\varnothing, \hat{p}^{[1]}_x, \hat{p}^{[1]}_y, \hat{p}^{[1]}_{xy}\right\} = \{1, p_x, p_y, p_x p_y\}.$

Probabilities $p_x = \mathrm{P}(x)$ and $p_y = \mathrm{P}(y)$ of two events $x$ and $y$ play for E-distribution of 2-plet of the events $\mathfrak{X} = \{x, y\}$ a role of parameters of the first order fixed in the theorem of A.Vorobyev [1]. Entropy of E-distribution of full 2-plet of the events $\mathfrak{X}$ coincides with E-distribution of its approximation of the second power: $H_{\hat{p}^{[2]}} = H_p = -\sum_{X \subseteq \mathfrak{X}} p(X) \ln p(X)$.

Entropy of E-distribution $p(X)$ concerning its multiplicative-truncated approximation $\hat{p}^{[N_0]}$ is defined by the formula (2), which after transformation comes to a kind

$$H_{p/\hat{p}^{[N_0]}} = -H_p - \sum_{X \subseteq \mathfrak{X}} p(X) \ln \hat{p}^{[N_0]}(X).$$

Let's consider relative entropy of E-distribution of a full 2-plet of events $\mathfrak{X}$:

$$H_{p/\hat{p}^{[0]}} = -H_p - \sum_{X \subseteq \mathfrak{X}} p(X) \ln\left(\tfrac{1}{4}\right) = -H_p + \ln 4 \sum_{X \subseteq \mathfrak{X}} p(X) = -H_p + H_{\hat{p}^{[0]}};$$

$$H_{p/\hat{p}^{[1]}} = -H_p - \sum_{X \subseteq \mathfrak{X}} p(X) \ln \hat{p}^{[1]}(X) = -H_p + \ln Z_1 - \ln \tau(x) \cdot [p(x) + p(x,y)] -$$

$$- \ln \tau(y) \cdot [p(y) + p(x,y)] = -H_p + \ln Z_1 - \ln \tau(x) \cdot p_x - \ln \tau(y) \cdot p_y = -H_p + \ln Z_1 -$$

$$- \ln \tau(x) \cdot \hat{p}_x^{[1]} - \ln \tau(y) \cdot \hat{p}_y^{[1]} = -H_p - \sum_{X \subseteq \mathfrak{X}} \hat{p}^{[1]}(X) \ln \hat{p}^{[1]}(X) = -H_p + H_{\hat{p}^{[1]}};$$

$$H_{p/\hat{p}^{[2]}} = \sum_{X \subseteq \mathfrak{X}} p(X) \ln \frac{p(X)}{\hat{p}^{[2]}(X)} = \sum_{X \subseteq \mathfrak{X}} p(X) \ln 1 = 0.$$

Multiplicative-truncated approximation, getting to a class of E-distributions with the fixed probabilities of *II* th sort, have the same probabilities of *II* th sort, as distribution: $\hat{p}_Y = p_Y$, $|Y| \le N_0$ [1], [3], [5], as the maximum of entropy of the E-distributions having fixed probabilities of *II* th sort on subsets of events to power $N_0$ inclusive, is reached on their multiplicative-truncated approximations of order $N_0$.

Thus, for any 2-plet of events $\mathfrak{X} = \{x, y\}$ is shown that $H_{p/\hat{p}^{[N_0]}} = -H_p + H_{\hat{p}^{[N_0]}}$.

Eventological theory of multicovariances and the theory of wide dependence are actively used for research an entropy properties in eventology. The results received for today are effectively applied in numerous appendices of eventology to modelling humanitarian and socio-economic systems [2].

## References

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