ON NON-CLASSIC STATEMENT OF GOURSAT FOUR-DIMENSIONAL PROBLEM FOR A PSEUDOPARABOLIC EQUATION

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At present, various classes of many-dimensional local and nonlocal boundary value problem [1-3] are developed by many mathematicians. This is connected with of these problems appearance in different applied character problems [4].

Problem statement. In the paper, the Goursat four-dimensional problem with non-classic conditions is substantiated for a pseudoparabolic equation .

Consider the equation:

$$\begin{aligned}(V_{1,1,2,2}u)(x) &\equiv D_1D_2D_3^2D_4^2u(x)+a_{1,0,2,2}(x)D_1D_3^2D_4^2u(x)+a_{0,1,2,2}(x)D_2D_3^2D_4^2u(x)+\\ &+a_{1,1,1,2}(x)D_1D_2D_3D_4^2u(x)+a_{1,1,2,1}(x)D_1D_2D_3^2D_4u(x)+\\ &+\sum_{\substack{i_1+i_2+i_3+i_4<5\\ i_\xi=\overline{0,1},\ \xi=\overline{1,2};\\ i\eta=\overline{0,2},\ \eta=\overline{3,4}}} a_{i_1,i_2,i_3,i_4}(x)D_1^{i_1}D_2^{i_2}D_3^{i_3}D_4^{i_4}u(x)=\varphi_{1,1,2,2}(x)\in L_p(G), \qquad (1)\end{aligned}$$

where $u(x)=u(x_1,x_2,x_3,x_4)$ is a desired function defined on G; $a_{i_1,i_2,i_3,i_4}=a_{i_1,i_2,i_3,i_4}(x)$ are the given measurable functions on $G=G_1\times G_2\times G_3\times G_4$, where $G_\xi=(0,h_\xi)$, $\xi=\overline{1,4}$; $\varphi_{1,1,2,2}(x)$ is a given measurable function on $G; D_\xi=\dfrac{\partial}{\partial x_\xi}$ is a generalized differentiation operator in S.L.Sobolev's sense.

In particular, this equation arises in studying the problems of fluid filtration in porous media, moisture transfer in soils, propagation of impulse radial waves, simulation of different biological processes phenomena, and also in the theory of optimal processes [5-7].

In this paper, equation (1) is investigated in the general case when the coefficients $a_{i_1,i_2,i_3,i_4}(x)$ are non-smooth functions satisfying only the following conditions:

$$a_{0,0,i_3,i_4}(x)\in L_p(G),\ \ a_{1,0,i_3,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,p,p,p}(G),\ a_{0,1,i_3,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{p,\infty,p,p}(G),\ a_{0,0,2,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{p,p,\infty,p}(G),$$

$$a_{0,0,i_3,2}(x)\in L^{x_1,x_2,x_3,x_4}_{p,p,p,\infty}(G),\ a_{1,1,i_3,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,\infty,p,p}(G),\ \ a_{1,0,2,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,p,\infty,p}(G),\ a_{1,0,i_3,2}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,p,p,\infty}(G),$$

$$a_{0,1,2,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{p,\infty,\infty,p}(G),a_{0,1,i_3,2}(x)\in L^{x_1,x_2,x_3,x_4}_{p,\infty,p,\infty}(G),\ \ a_{0,0,2,2}(x)\in L^{x_1,x_2,x_3,x_4}_{p,p,\infty,\infty}(G),a_{1,1,2,i_4}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,\infty,\infty,p}(G),$$

$$a_{1,1,i_3,2}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,\infty,p,\infty}(G),\ a_{0,1,2,2}(x)\in L^{x_1,x_2,x_3,x_4}_{p,\infty,\infty,\infty}(G),\ \ a_{1,0,1,1}(x)\in L^{x_1,x_2,x_3,x_4}_{\infty,p,\infty,\infty}(G),$$

where $i_3=\overline{0,1},\ i_4=\overline{0,1}.$

Under these conditions, the solution $u(x)$ of equation (1) will be sought in S.L.Sobolev's space $W_p^{(1,1,2,2)}(G)\equiv\{u(x):\ D_1^{i_1}D_2^{i_2}D_3^{i_3}D_4^{i_4}u(x)\in L_p(G),\ \ i_\xi=\overline{0,1},\ \xi=\overline{1,2};\ i_\eta=\overline{0,2},\ \eta=\overline{3,4}\}$, where $1\le p\le\infty$. We'll define the norm in the anisotropic space $W_p^{(1,1,2,2)}(G)$ by the equality

$$\|u(x)\|_{W_p^{(1,1,2,2)}(G)}=\sum_{\substack{i_\xi=0\\ \xi=\overline{1,2}}}^{1}\ \sum_{\substack{i_\eta=0\\ \eta=\overline{3,4}}}^{2}\left\|D_1^{i_1}D_2^{i_2}D_3^{i_3}D_4^{i_4}u(x)\right\|_{L_p(G)}.$$

For equation (1), we can give the classic type Goursat condition in the form

$$\begin{cases} u(x_1,x_2,x_3,x_4)\big|_{x_1=0} = F(x_2,x_3,x_4); \quad u(x_1,x_2,x_3,x_4)\big|_{x_2=0} = g(x_1,x_3,x_4); \\ u(x_1,x_2,x_3,x_4)\big|_{x_3=0} = \psi(x_1,x_2,x_4); \quad \left.\dfrac{\partial u(x_1,x_2,x_3,x_4)}{\partial x_3}\right|_{x_3=0} = \Phi(x_1,x_2,x_4); \\ u(x_1,x_2,x_3,x_4)\big|_{x_4=0} = T(x_1,x_2,x_3); \quad \left.\dfrac{\partial u(x_1,x_2,x_3,x_4)}{\partial x_4}\right|_{x_4=0} = S(x_1,x_2,x_3). \end{cases} \tag{2}$$

where $F(x_2,x_3,x_4), g(x_1,x_3,x_4), \psi(x_1,x_2,x_4), \Phi(x_1,x_2,x_4), T(x_1,x_2,x_3), S(x_1,x_2,x_3)$, are the given measurable functions on G. It is obvious that in the case of conditions (2) the functions F, g, ψ, Φ, T, S in addition to the conditions

$$F(x_2,x_3,x_4) \in W_p^{(1,2,2)}(G_2\times G_3\times G_4), \quad g(x_1,x_3,x_4) \in W_p^{(1,2,2)}(G_1\times G_3\times G_4),$$
$$\psi(x_1,x_2,x_4) \in W_p^{(1,1,2)}(G_1\times G_2\times G_4), \quad \Phi(x_1,x_2,x_4) \in W_p^{(1,1,2)}(G_1\times G_2\times G_4),$$
$$T(x_1,x_2,x_3) \in W_p^{(1,1,2)}(G_1\times G_2\times G_3), \quad S(x_1,x_2,x_3) \in W_p^{(1,1,2)}(G_1\times G_2\times G_3),$$

should also satisfy the following conditions:

$$\begin{cases} F(0,x_3,x_4) = g(0,x_3,x_4); \; F(x_2,0,x_4) = \psi(0,x_2,x_4); \; g_{x_4}(x_1,x_3,0) = S(x_1,0,x_3); \\ F(x_2,x_3,0) = T(0,x_2,x_3); \; F_{x_3}(x_2,0,x_4) = \Phi(0,x_2,x_4); \; F_{x_4}(x_2,x_3,0) = S(0,x_2,x_3); \\ g(x_1,x_3,0) = T(x_1,0,x_3); \; g(x_1,0,x_4) = \psi(x_1,0,x_4); \; g_{x_3}(x_1,0,x_4) = \Phi(x_1,0,x_4); \\ \psi(x_1,x_2,0) = T(x_1,x_2,0); \; \psi_{x_4}(x_1,x_2,0) = S(x_1,x_2 0,); \; \Phi_{x_4}(x_1,x_2,0) = S_{x_3}(x_1,x_2,0), \end{cases} \tag{3}$$

that are agreement conditions.

Availability of agreement conditions in the statement of problem (1), (2) means that some conditions (2) give also unnecessary information on the solution of this problem. Therefore, there arises a question on finding boundary conditions that don't contain unnecessary information on the solution and don't require fulfillment of some agreement type additional conditions. In this connection, we consider the following non-classic initial-boundary conditions:

$$V_{0,0,i_3,i_4}u \equiv D_3^{i_3}D_4^{i_4}u(0,0,0,0) = \varphi_{0,0,i_3,i_4} \in R, \quad i_\nu = \overline{0,1}, \quad \nu = \overline{3,4};$$
$$(V_{1,0,i_3,i_4}u)(x_1) \equiv D_1D_3^{i_3}D_4^{i_4}u(x_1,0,0,0) = \varphi_{1,0,i_3,i_4}(x_1) \in L_p(G_1), \quad i_\nu = \overline{0,1}, \quad \nu = \overline{3,4};$$
$$(V_{0,1,i_3,i_4}u)(x_2) \equiv D_2D_3^{i_3}D_4^{i_4}u(0,x_2,0,0) = \varphi_{0,1,i_3,i_4}(x_2) \in L_p(G_2), \quad i_\nu = \overline{0,1}, \quad \nu = \overline{3,4};$$
$$(V_{0,0,2,i_4}u)(x_3) \equiv D_3^2D_4^{i_4}u(0,0,x_3,0) = \varphi_{0,0,2,i_4}(x_3) \in L_p(G_3), \; i_4 = \overline{0,1};$$
$$(V_{0,0,i_3,2}u)(x_4) \equiv D_3^{i_3}D_4^2u(0,0,0,x_4) = \varphi_{0,0,i_3,2}(x_4) \in L_p(G_4), \; i_3 = \overline{0,1};$$
$$(V_{1,1,i_3,i_4}u)(x_1,x_2) \equiv D_1D_2D_3^{i_3}D_4^{i_4}u(x_1,x_2,0,0) = \varphi_{1,1,i_3,i_4}(x_1,x_2) \in L_p(G_1\times G_2), \; i_\nu = \overline{0,1}, \; \nu = \overline{3,4};$$
$$(V_{1,0,2,i_4}u)(x_1,x_3) \equiv D_1D_3^2D_4^{i_4}u(x_1,0,x_3,0) = \varphi_{1,0,2,i_4}(x_1,x_3) \in L_p(G_1\times G_3), \quad i_4 = \overline{0,1};$$
$$(V_{1,0,i_3,2}u)(x_1,x_4) \equiv D_1D_3^{i_3}D_4^2u(x_1,0,0,x_4) = \varphi_{1,0,i_3,2}(x_1,x_4) \in L_p(G_1\times G_4), \quad i_3 = \overline{0,1};$$
$$(V_{0,1,2,i_4}u)(x_2,x_3) \equiv D_2D_3^2D_4^{i_4}u(0,x_2,x_3,0) = \varphi_{0,1,2,i_4}(x_2,x_3) \in L_p(G_2\times G_3), \quad i_4 = \overline{0,1};$$
$$(V_{0,1,i_3,2}u)(x_2,x_4) \equiv D_2D_3^{i_3}D_4^2u(0,x_2,0,x_4) = \varphi_{0,1,i_3,2}(x_2,x_4) \in L_p(G_2\times G_4), \quad i_3 = \overline{0,1};$$
$$(V_{0,0,2,2}u)(x_3,x_4) \equiv D_3^2D_4^2u(0,0,x_3,x_4) = \varphi_{0,0,2,2}(x_3,x_4) \in L_p(G_3\times G_4);$$
$$(V_{1,1,2,i_4}u)(x_1,x_2,x_3) \equiv D_1D_2D_3^2D_4^{i_4}u(x_1,x_2,x_3,0) = \varphi_{1,1,2,i_4}(x_1,x_2,x_3) \in L_p(G_1\times G_2\times G_3), \; i_4 = \overline{0,1};$$
$$(V_{1,1,i_3,2}u)(x_1,x_2,x_4) \equiv D_1D_2D_3^{i_3}D_4^2u(x_1,x_2,0,x_4) = \varphi_{1,1,i_3,2}(x_1,x_2,x_4) \in L_p(G_1\times G_2\times G_4), \; i_3 = \overline{0,1};$$

$$(V_{0,1,2,2}u)(x_2,x_3,x_4)\equiv D_2D_3^2D_4^2u(0,x_2,x_3,x_4)=\varphi_{0,1,2,2}(x_2,x_3,x_4)\in L_p(G_2\times G_3\times G_4);$$
$$(V_{1,0,2,2}u)(x_1,x_3,x_4)\equiv D_1D_3^2D_4^2u(x_1,0,x_3,x_4)=\varphi_{1,0,2,2}(x_1,x_3,x_4)\in L_p(G_1\times G_3\times G_4); \qquad (4)$$

If the function $u\in W_p^{(1,1,2,2)}(G)$ is a solution of four-dimensional classic type Goursat problem (1), (2), then it is also a solution of problem (1), (4) for $\varphi_{i_1,i_2,i_3,i_4}$ determined by the equalities

$$\varphi_{0,0,0,0}=F(0,0,0)=g(0,0,0)=\psi(0,0,0)=T(0,0,0);\ \ \varphi_{0,0,1,0}=\Phi(0,0,0)=g_{x_3}(0,0,0)=F_{x_3}(0,0,0);$$
$$\varphi_{0,0,0,1}=S(0,0,0)=F_{x_4}(0,0,0)=\psi_{x_4}(0,0,0);\ \varphi_{0,0,1,1}=S_{x_3}(0,0,0)=\Phi_{x_4}(0,0,0);$$
$$\varphi_{1,0,0,0}(x_1)=g_{x_1}(x_1,0,0)=\psi_{x_1}(x_1,0,0)=T_{x_1}(x_1,0,0);\varphi_{1,0,1,0}(x_1)=g_{x_1x_3}(x_1,0,0)=\Phi_{x_1}(x_1,0,0)=T_{x_1x_3}(x_1,0,0);$$
$$\varphi_{1,0,0,1}(x_1)=g_{x_1x_4}(x_1,0,0)=\psi_{x_1x_4}(x_1,0,0)=S_{x_1}(x_1,0,0);\ \varphi_{1,0,1,1}(x_1)=g_{x_1x_3x_4}(x_1,0,0)=\Phi_{x_1x_4}(x_1,0,0);$$
$$\varphi_{0,1,0,0}(x_2)=F_{x_2}(x_2,0,0)=\psi_{x_2}(0,x_2,0)=T_{x_2}(0,x_2,0);\varphi_{0,1,1,0}(x_2)=F_{x_2x_3}(x_2,0,0)=T_{x_2x_3}(0,x_2,0)=\Phi_{x_2}(0,x_2,0);$$
$$\varphi_{0,1,0,1}(x_2)=F_{x_2x_4}(x_2,0,0)=\psi_{x_2x_4}(0,x_2,0)=S_{x_2}(0,x_2,0);\ \varphi_{0,1,1,1}(x_2)=F_{x_2x_3x_4}(x_2,0,0)=S_{x_2x_3}(0,x_2,0);$$
$$\varphi_{0,0,2,0}(x_3)=F_{x_3x_3}(0,x_3,0)=g_{x_3x_3}(0,x_3,0)=T_{x_3x_3}(0,0,x_3);$$
$$\varphi_{0,0,2,1}(x_3)=F_{x_3x_3x_4}(0,x_3,0)=g_{x_3x_3x_4}(0,x_3,0)=S_{x_3x_3}(0,0,x_3);$$
$$\varphi_{0,0,0,2}(x_4)=F_{x_4x_4}(0,0,x_4)=g_{x_4x_4}(0,0,x_4)=\psi_{x_4x_4}(0,0,x_4);$$
$$\varphi_{0,0,1,2}(x_4)=F_{x_3x_4x_4}(0,0,x_4)=g_{x_3x_4x_4}(0,0,x_4)=\Phi_{x_4x_4}(0,0,x_4);$$
$$\varphi_{1,1,0,0}(x_1,x_2)=\psi_{x_1x_2}(x_1,x_2,0)=T_{x_1x_2}(x_1,x_2,0);\ \ \varphi_{1,1,1,0}(x_1,x_2)=T_{x_1x_2x_3}(x_1,x_2,0)=\Phi_{x_1x_2}(x_1,x_2,0);$$
$$\varphi_{1,1,0,1}(x_1,x_2)=\psi_{x_1x_2x_4}(x_1,x_2,0)=S_{x_1x_2}(x_1,x_2,0);\ \ \varphi_{1,1,1,1}(x_1,x_2)=\Phi_{x_1x_2x_4}(x_1,x_2,0)=S_{x_1x_2x_3}(x_1,x_2,0);$$
$$\varphi_{1,0,2,0}(x_1,x_3)=g_{x_1x_3x_3}(x_1,x_3,0)=T_{x_1x_3x_3}(x_1,0,x_3);\ \ \varphi_{1,0,2,1}(x_1,x_3)=g_{x_1x_3x_3x_4}(x_1,x_3,0)=S_{x_1x_3x_3}(x_1,0,x_3);$$
$$\varphi_{1,0,0,2}(x_1,x_4)=g_{x_1x_4x_4}(x_1,0,x_4)=\psi_{x_1x_4x_4}(x_1,0,x_4);\ \varphi_{1,0,1,2}(x_1,x_4)=g_{x_1x_3x_4x_4}(x_1,0,x_4)=\Phi_{x_1x_4x_4}(x_1,0,x_4);$$
$$\varphi_{0,1,2,0}(x_2,x_3)=F_{x_2x_3x_3}(x_2,x_3,0)=T_{x_2x_3x_3}(0,x_2,x_3);\varphi_{0,1,2,1}(x_2,x_3)=F_{x_2x_3x_3x_4}(x_2,x_3,0)=S_{x_2x_3x_3}(0,x_2,x_3);$$
$$\varphi_{0,1,0,2}(x_2,x_4)=F_{x_2x_4x_4}(x_2,0,x_4)=\psi_{x_2x_4x_4}(0,x_2,x_4);\varphi_{0,1,1,2}(x_2,x_4)=F_{x_2x_3x_4x_4}(x_2,0,x_4)=\Phi_{x_2x_4x_4}(0,x_2,x_4);$$
$$\varphi_{0,0,2,2}(x_3,x_4)=F_{x_3x_3x_4x_4}(0,x_3,x_4)=g_{x_3x_3x_4x_4}(0,x_3,x_4);\ \varphi_{1,1,2,0}(x_1,x_2,x_3)=T_{x_1x_2x_3x_3}(x_1,x_2,x_3);$$
$$\varphi_{1,1,2,1}(x_1,x_2,x_3)=S_{x_1x_2x_3x_3}(x_1,x_2,x_3);\ \ \varphi_{1,1,0,2}(x_1,x_2,x_4)=\psi_{x_1x_2x_4x_4}(x_1,x_2,x_4);$$
$$\varphi_{1,1,1,2}(x_1,x_2,x_4)=\Phi_{x_1x_2x_4x_4}(x_1,x_2,x_4);\ \ \varphi_{0,1,2,2}(x_2,x_3,x_4)=F_{x_2x_3x_3x_4x_4}(x_2,x_3,x_4);$$
$$\varphi_{1,0,2,2}(x_1,x_3,x_4)=g_{x_1x_3x_3x_4x_4}(x_1,x_3,x_4).$$

It is easy to prove that the contrary one is also true. In other words if the function $u\in W_p^{(1,1,2,2)}(G)$ is a solution of problem (1), (4), it is also a solution of problem (1), (2), for the functions $F,\ g,\ \psi,\ \Phi,\ T,\ S$:

$$F(x_2,x_3,x_4)=\sum_{i_3=0}^{1}\sum_{i_4=0}^{1}x_3^{i_3}x_4^{i_4}\varphi_{0,0,i_3,i_4}+\sum_{i_3=0}^{1}\sum_{i_4=0}^{1}x_3^{i_3}x_4^{i_4}\int_0^{x_2}\varphi_{0,1,i_3,i_4}(\xi_2)d\xi_2+\sum_{i_4=0}^{1}x_4^{i_4}\int_0^{x_3}(x_3-\xi_3)\varphi_{0,0,2,i_4}(\xi_3)d\xi_3+$$
$$+\sum_{i_3=0}^{1}x_3^{i_3}\int_0^{x_4}(x_4-\xi_4)\varphi_{0,0,i_3,2}(\xi_4)d\xi_4+\sum_{i_4=0}^{1}x_4^{i_4}\int_0^{x_2}\int_0^{x_3}(x_3-\xi_3)\varphi_{0,1,2,i_4}(\xi_2,\xi_3)d\xi_2d\xi_3+$$
$$+\int_0^{x_3}\int_0^{x_4}(x_3-\xi_3)(x_4-\xi_4)\varphi_{0,0,2,2}(\xi_3,\xi_4)d\xi_3d\xi_4+\sum_{i_3=0}^{1}x_3^{i_3}\int_0^{x_2}\int_0^{x_4}(x_4-\xi_4)\varphi_{0,1,i_3,2}(\xi_2,\xi_4)d\xi_2d\xi_4+$$
$$+\int_0^{x_2}\int_0^{x_3}\int_0^{x_4}(x_3-\xi_3)(x_4-\xi_4)\varphi_{0,1,2,2}(\xi_2,\xi_3,\xi_4)d\xi_2d\xi_3d\xi_4\ ;$$

$$g(x_1,x_3,x_4)=\sum_{i_3=0}^{1}\sum_{i_4=0}^{1}x_3^{i_3}x_4^{i_4}\varphi_{0,0,i_3,i_4}+\sum_{i_3=0}^{1}\sum_{i_4=0}^{1}x_3^{i_3}x_4^{i_4}\int_0^{x_1}\varphi_{1,0,i_3,i_4}(\xi_1)d\xi_1+\sum_{i_4=0}^{1}x_4^{i_4}\int_0^{x_3}(x_3-\xi_3)\varphi_{0,0,2,i_4}(\xi_3)d\xi_3+$$
$$+\sum_{i_3=0}^{1}x_3^{i_3}\int_0^{x_4}(x_4-\xi_4)\varphi_{0,0,i_3,2}(\xi_4)d\xi_4+\sum_{i_4=0}^{1}x_4^{i_4}\int_0^{x_1}\int_0^{x_3}(x_3-\xi_3)\varphi_{1,0,2,i_4}(\xi_1,\xi_3)d\xi_1d\xi_3+$$

$$+\int_0^{x_3}\int_0^{x_4}(x_3-\xi_3)(x_4-\xi_4)\varphi_{0,0,2,2}(\xi_3,\xi_4)d\xi_3 d\xi_4+\sum_{i_3=0}^{1}x_3^{i_3}\int_0^{x_1}\int_0^{x_4}(x_4-\xi_4)\varphi_{1,0,i_3,2}(\xi_1,\xi_4)d\xi_1 d\xi_4+$$

$$+\int_0^{x_1}\int_0^{x_3}\int_0^{x_4}(x_3-\xi_3)(x_4-\xi_4)\varphi_{1,0,2,2}(\xi_1,\xi_3,\xi_4)d\xi_1 d\xi_3 d\xi_4\ ;$$

$$\psi(x_1,x_2,x_4)=M_0(x_1,x_2,x_4),\ \Phi(x_1,x_2,x_4)=M_1(x_1,x_2,x_4),\text{ where}$$

$$M_k(x_1,x_2,x_4)=\varphi_{0,0,k,0}+x_4\varphi_{0,0,k,1}+\int_0^{x_1}\varphi_{1,0,k,0}(\tau_1)d\tau_1+x_4\int_0^{x_1}\varphi_{1,0,k,1}(\tau_1)d\tau_1+\int_0^{x_2}\varphi_{0,1,k,0}(\tau_2)d\tau_2+$$

$$+x_4\int_0^{x_2}\varphi_{0,1,k,1}(\tau_2)d\tau_2+\int_0^{x_4}(x_4-\tau_4)\varphi_{0,0,k,2}(\tau_4)d\tau_4+\int_0^{x_1}\int_0^{x_2}\varphi_{1,1,k,0}(\tau_1,\tau_2)d\tau_1 d\tau_2+x_4\int_0^{x_1}\int_0^{x_2}\varphi_{1,1,k,1}(\tau_1,\tau_2)d\tau_1 d\tau_2+$$

$$+\int_0^{x_2}\int_0^{x_4}(x_4-\tau_4)\varphi_{0,1,k,2}(\tau_2,\tau_4)d\tau_2 d\tau_4+\int_0^{x_1}\int_0^{x_4}(x_4-\tau_4)\varphi_{1,0,k,2}(\tau_1,\tau_4)d\tau_1 d\tau_4+$$

$$+\int_0^{x_1}\int_0^{x_2}\int_0^{x_4}(x_4-\tau_4)\varphi_{1,1,k,2}(\tau_1,\tau_2,\tau_4)d\tau_1 d\tau_2 d\tau_4,\quad k=\overline{0,1}\ ;$$

$$T(x_1,x_2,x_3)=L_0(x_1,x_2,x_3),\ \ S(x_1,x_2,x_3)=L_1(x_1,x_2,x_3),\text{ where}$$

$$L_k(x_1,x_2,x_3)=\varphi_{0,0,0,k}+x_3\varphi_{0,0,1,k}+\int_0^{x_1}\varphi_{1,0,0,k}(\eta_1)d\eta_1+x_3\int_0^{x_1}\varphi_{1,0,1,k}(\eta_1)d\eta_1+\int_0^{x_2}\varphi_{0,1,0,k}(\eta_2)d\eta_2+$$

$$+x_3\int_0^{x_2}\varphi_{0,1,1,k}(\eta_2)d\eta_2+\int_0^{x_3}(x_3-\eta_3)\varphi_{0,0,2,k}(\eta_3)d\eta_3+\int_0^{x_1}\int_0^{x_2}\varphi_{1,1,0,k}(\eta_1,\eta_2)d\eta_1 d\eta_2+$$

$$+x_3\int_0^{x_1}\int_0^{x_2}\varphi_{1,1,1,k}(\eta_1,\eta_2)d\eta_1 d\eta_2+\int_0^{x_2}\int_0^{x_3}(x_3-\eta_3)\varphi_{0,1,2,k}(\eta_2,\eta_3)d\eta_2 d\eta_3+$$

$$+\int_0^{x_1}\int_0^{x_3}(x_3-\eta_3)\varphi_{1,0,2,k}(\eta_1,\eta_3)d\eta_1 d\eta_3+\int_0^{x_1}\int_0^{x_2}\int_0^{x_3}(x_3-\eta_3)\varphi_{1,1,2,k}(\eta_1,\eta_2,\eta_3)d\eta_1 d\eta_2 d\eta_3,\quad k=\overline{0,1}\,.$$

Thus, Goursat classic type four-dimensional problems of type (1), (2) and of type (1), (4) are equivalent in the general case. However, Goursat four-dimensional problems (1), (4) is more natural in statement than problem (1), (2). This is connected with the fact that no agreement type additional conditions on the right hand sides of boundary conditions are required in the statement of problem (1), (4). Therefore, problem (1), (4) may be considered as the Goursat problem with non-classic conditions.

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