OPTIMIZATION OF TRANSIENT PROCESSES IN OIL AND GAS PIPELINES

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We set out the statement and analysis of the results of numerical investigation of transient states in trunk pipelines in the presence of optimal quick-action boundary control by the transient process arising when switching from one steady-state behavior of raw material transportation to another.

The problem considered is connected with a control problem for wave process that was studied by a series of scientists including Lions, Ilyin, Butkovski, Vasilyev etc. [2-4, 6].

In contrast to the investigations carried out before, in the given work, we investigate a problem of optimal quick-action boundary control by the regimes of fluid transportation (oil) in pipelines in the presence of constraints of technological character imposed on the control actions and on the state of the controlled object. We give a qualitative analysis of the dependence of the minimal transient-process time from the dissipation coefficient, the length of the pipeline, the difference between the values of initial and final steady-state regimes, and the range of admissible controls for different values of initial and final steady-state regimes. We consider the controls on the class of piecewise continuous and of piecewise constant functions. In the latter case the moments of controls' switchings are also optimized.

Consider an isothermal transportation process of one-phase oil flow over a linear part of a horizontal pipeline of length l, of diameter d, and of the coefficient of hydraulic resistance λ. The regime of fluid flow is assumed to be laminar; oil is assumed to be incompressible having kinematical viscosity ν. On both ends of the oil pipeline, there are pumping stations providing the given transit regime.

Unsteady laminar flow of incompressible fluid with constant density for practical purposes is sufficiently adequately described by the following linearized system of differential equations [1, p.15]:

$$-\frac{\partial p}{\partial x} = \frac{\partial \omega}{\partial t} + 2a\omega, \quad -\frac{\partial p}{\partial t} = c^2 \frac{\partial \omega}{\partial x}, \tag{1}$$

where $p = p(x,t), \quad \omega = \omega(x,t)$ are correspondingly fluid's density and flow velocity at the point of the pipeline $x \in (0,l)$ at the moment of time $t > t_0$, c is the acoustic sound velocity in the environment, λ is the coefficient of hydraulic resistance, $2a = \lambda\omega / 2d = 32\nu/d^2$.

Suppose that up until some moment of time t_0 there had been a steady-state regime defined by the conditions

$$\omega(x,t) = \omega_0, \quad p(x,t) = p_0(x), \; x \in [0,l], \, t \le t_0, \tag{2}$$

where the known function $p_0(x)$ at the given fluid flow velocity ω_0 is defined by the geometrical dimensions of the pipeline and by the properties of the fluid (oil).

It is necessary to note that usually in practice it is difficult to satisfy conditions (2) precisely, since there are always small perturbations in the pipeline caused by the certain non-rhythmicity of the work of a technological equipment leading to moderate deviations from the establishment conditions (2):

$$\begin{aligned} &\left|\omega(x,t) - \omega_0\right| \le \delta_\omega, \\ &\left|p(x,t) - p_0(x)\right| \le \delta_p, \quad x \in (0,1), \quad t \le t_0, \end{aligned} \tag{3}$$

where δ_ω, δ_p are positive small values measured in percentages correspondingly from the values ω_0 and $p_0(x)$ of some steady-state regime depending, as a rule, from the precision of measuring technique used in the system of control by pipeline transportation.

In this connection under δ-steady-state regime we understand such regime of raw material transportation over the pipeline at which conditions (3) are satisfied.

Conditions (2) are satisfied at the expense of pumping stations maintaining the regime

$$\omega(0,t) = \omega(l,t) = \omega_0, \quad t \le t_0. \tag{4}$$

Suppose that it is necessary to switch the pipeline to a new steady-state regime under satisfying the following conditions

$$\omega(x,t) = \omega_T,\ p(x,t) = p_T(x),\ t \ge T, x \in [0,l], \tag{5}$$

where T is the time at which a new steady-state regime (7) would commence proceeding.

The necessary change of the transportation regimes in pipelines must be provided by the changes in the regimes of pumping stations' work, namely, at the expense of the change of volumetric expenditure (that is equivalent to the change in raw material's flow velocity) at the ends of the linear part of the pipeline

$$\omega(0,t) = u_1(t), \qquad \omega(l,t) = u_2(t), \qquad t \in [t_0, T], \tag{6}$$

under the condition of the fulfillment of some technological and technical constraints (taking into account the characteristics of the pumps):

$$\underline{u}_1 \le u_1(t) \le \overline{u}_1, \qquad \underline{u}_2 \le u_2(t) \le \overline{u}_2, \qquad t \in [t_0, T], \tag{7}$$

where $u_1(t), u_2(t)$ are piecewise continuous functions.

While controlling real technological processes, including the regimes of raw material pipeline transportation, the implementation of control actions on the class of piecewise continuous functions is often complex or even impossible. That is why on practice they consider control problems on technically easily implemented classes of functions such as piecewise constant, impulse etc. In this connection, in the given work, we also consider a class of boundary control problems for process (1) when the controls actions are the functions of the kind:

$$\begin{aligned} &u_i(t) = v_{ij} = const,\ t \in [\tau_{j-1}, \tau_j),\ i = 1,2,\ j = 1,...,L \\ &\tau_0 = 0, \quad \tau_L = T, \quad \tau_j = \tau_{j-1} + \Delta\tau_j,\ j = \overline{1, L-1}. \end{aligned} \tag{8}$$

In this case, the optimal control problem consists in determining the finite-dimensional vectors $v_1, v_2 \in E^L$. As for the moments of time τ_j, and correspondingly the intervals $\Delta\tau_j$, they may be determined by many ways. [At the given number of switchings L of the control actions, there may be cases when switching times τ_j or $\Delta\tau_j = \Delta\tau = const, j = \overline{1, L}$ are given. The moments of switching times τ_j, $j = \overline{1, L-1}$ can be optimized as well. In this case, the optimized vector is ($v_{11},..., v_{L1}, v_{12},..., v_{L2}, \tau_1,..., \tau_{L-1}$).

In the present work, the results of the solution of optimal control problems for transient processes are given. We assume that L is given, and both the values of piecewise constant controls on their constancy intervals and the moments of control switchings τ_j are optimized.

Reasoning from the pipeline strength conditions, it is necessary to meet the following technological constraint on the magnitude of the maximal value of pressure while considering the transportation process over the whole pipeline and during the whole period of controlling the transient process

$$\underline{p} \le p(x,t) \le \overline{p}, \qquad x \in (0,1), \qquad t \in [t_0, T], \tag{9}$$

where $\overline{p}$ is the given overload capacity of pressure depending from the properties of the material, which the pipeline is made of; $\underline{p}$ is the magnitude of the pressure (also called cavitational resource), below which the undesirable cavitation process (oil boil) occurs.

Constraints (9) can be transformed into constraints on the overload capacity of the linear velocity $\overline{\omega}$ along the whole length of the pipeline and during the whole interval of the control by the process:

$$\underline{\omega} \le \omega(x,t) \le \overline{\omega}, \qquad x \in (0,l), \qquad t \in [t_0, T], \tag{10}$$

Taking into account previous considerations about δ-steady-state regime we take as the target functional the following functional:

$$J(u,T) = T + \int_T^{T+DT} \int_0^l \left\{ r_1 [p(x,t) - p_T(x)]^2 + r_2 [\omega(x,t) - \omega_T]^2 \right\} dxdt +$$

$$+ R_1 \int_0^T \int_0^l \left[\max(0, \omega(x,t) - \overline{\omega})\right]^2 dxdt + R_2 \int_0^T \int_0^l \left[\max(0, -\omega(x,t) + \underline{\omega})\right]^2 dxdt \to \min \tag{11}$$

Here DT is beforehand given length of the time interval, on which observation over the process of transit and of establishment of the presence of δ-steady-state regime is carried out.

The stated problem can be considered as a problem of optimal quick-action control by a distributed system at the given values of the state functions at the indefinite moment of completion time T (considered as optimized) and with control in boundary conditions. To solve the problem, two approaches are proposed. According to the first approach, we can consider T as a parameter and use two-level optimization: on the upper level, in order to determine the optimal time of carrying on transient process T^* we apply any one-dimensional method; on the lower level, at the given current values of T, in order to determine

$$J_T^{\ *} = J(u_T^*, T) = \min_u J(u,T)$$

we solve the optimal control problem for a distributed system with a fixed time. [according to the second approach, T is considered as a component of the control; to determine the optimal value of this component we apply a joint optimization both on T and $u(t)$.

For both approaches, necessary optimality conditions are obtained. They contain formulas for the components of the target functional on the optimized parameters – the operating regimes of the pumping stations and the completion time of the process T. The obtained formulas allow to use efficient numerical methods of first order optimization [5] for the solution to the optimal control problems.

Various computational experiments have been carried out in the aim of disclosing some regularities in the dependence of the minimal transient-process time from the length of a part of the pipeline, from the dissipation coefficient a, and from the difference in the values of the parameters of the initial and final steady-state regimes. We investigated the control problem for transient processes without any constraints on the state and control functions, as well as with operating and technological constraints (9), (10).

While carrying out numerical experiments under the constraints on the values of the functions of control actions and of state, we disclosed that the transient period of the optimal transient process depends on the length of the interval of admissible values of the controls $[\underline{u}, \overline{u}]$ for the given values of the initial and final steady-state regimes.

The control was considered on classes of piecewise continuous and constant functions. We also optimized the switching periods $\tau_j, j = \overline{1, L-1}$ of the controls when the value L was given while considering classes of piecewise constant functions.

The summary of the results of numerical experiments is as follows.

1. As is well known from theoretical investigations, which have been certified by the numerical experiments carried out, the minimal transient-process time in the presence of piecewise continuous control actions without any constraints on the control process does not depend on the diameter of a pipeline, on coefficient of resistance, on viscosity, on oil density, and on the values of initial and final steady-state regimes. The minimal transient-process time does depend on the length of a pipeline. But the optimal regimes of pumping stations in this case are practically unrealizable because of the big oscillation of the control functions.

2. In the presence of technological constraints on the magnitude of the range (boundaries) of the control actions from a class of piecewise continuous functions, there take place the following facts:

2.1. The dissipation coefficient has an influence on the transient-process time, namely, when the dissipation coefficient increases, the transient period decreases. At that the larger the range of the set of admissible controls, the less the influence of the dissipation coefficient.

2.2. The difference between initial and final steady-state regimes has an influence on the transient-process time, namely, the more the consumption, the longer the transient-process time. The influence of the difference on the transient period decreases (up to zero) when the range of the set of admissible values of the control actions is increased.

2.3. Transient-process time increases when the length of a part of the pipeline increases.

2.4. The minimal transient-process time when switching from a less value of the regime to a larger one and vice-versa is the same. At that the optimal switching regimes are symmetric.

3. When controlling the transient process on a class of piecewise constant functions in the presence of technological constraints on the controls' regimes, all qualitative characteristics 2.1-2.4 possessed by piecewise constant regimes are observed in this class too.

3.1. In comparison to piecewise constant controls, here more time is required for the transient process in the presence of the same initial values of the technological parameters.

3.2. When the number of constancy intervals and the range of admissible values of the controls increase the transient-process time decreases.

3.3. In case when the moments of controls' switchings are also optimized, the decrease of the range of admissible values of the control results in the decrease of the number of controls' constancy intervals.

As the numerical experiments show, when controlling transient processes on a part of the oil pipeline at the expense of pumping stations located at both its ends, the transient period decreases twice in comparison with controlling a pumping station at one end, irrespective of a class of control actions and the range of admissible controls.

In the general case, it is impossible to transfer the given above qualitative analysis on basis of computer-based experiments into quantitative assessments. But for every specific case of an oil pipeline and of the characteristics of oil, we can obtain quantitative characteristics of transient processes and recommendations on how to control them in the form of graphs, tables, and even more specific technological recommendations at the expense of carrying out numerous experiments

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