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NECESSARY OPTIMALITY CONDITIONS IN ONE NON-SMOOTH OPTIMAL CONTROL PROBLEM WITH VARIABLE STRUCTURE

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Assume that it is required to minimize the functional

$$J(u,v)=\max_{z\in Z}\varphi(y(t_2),z), \tag{1}$$

under constraints

$$\begin{aligned} &u(t)\in U\subset R^r, \quad t\in[t_0,t_1],\\ &v(t)\in V\subset R^q, \quad t\in[t_1,t_2], \end{aligned} \tag{2}$$

$$\begin{aligned} &\dot{x}=f(t,x,u), \quad t\in[t_0,t_1], \quad x(t_0)=x_0\\ &\dot{y}=g(t,y,v), \quad t\in[t_1,t_2], \end{aligned} \tag{3}$$

$$y(t_1)=G(x(t_1)), \tag{4}$$

$$\Phi_k(y(t_2))=0\,,\ k=\overline{1,p}\,. \tag{5}$$

Here, $Z\subset R^\ell$ is a compact set of ℓ-dimensional vectors z, $f(t,x,u)$ $(g(t,y,v))$ is a given n (m) - dimensional vector-function continuous in $[t_0,t_1]\times R^n\times R^r$ $([t_1,t_2]\times R^m\times R^v)$ together with partial derivatives with respect to x (y), $G(x)$ is a continuously differentiable m-dimensional vector-function given in R^n, $\varphi(y,z)$ is a given scalar function continuous in $R^m\times Z$ together with partial derivatives with respect to y, $u(t)$ $(v(t))$ is r (q)-dimensional piecewise – continuous vector of control actions with values from the given non-smooth and bounded set U (V), $\Phi_k(y)$, $k=\overline{1,p}$ are the given continuously differentiable in R^m scalar functions, moreover, the Jacobian $\left\|\frac{\partial\Phi_k}{\partial x_i}\right\|$ has its own maximal rank p.

Assume

$$W=\{(u(t),v(t)): u(t)\in U\subset R^r,\ t\in[t_0,t_1],\ v(t)\in V\subset R^q,\ t\in[t_1,t_2];\ \Phi_k(y(t_2))=0,\ k=\overline{1,p}\}.$$

The set W is said to be a set of admissible controls.

Let by the definition

$$R(u^0,v^0)=\{z\in Z: S(u^0,v^0)=\varphi(y^0(t_2),z)\}.$$

The admissible control $(u^0(t),\, v^0(t))$ is said to be an optimal control if

$$S(u^0,v^0)=\min_{\substack{u\in U\\ v\in V}} S(u,v).$$

Assuming $(u^0(t),v^0(t))$ an optimal control, we define the "perturbed" control by the formula

$$\begin{cases} u_\varepsilon(t)=\begin{cases} u^0(t),\ t\,\overline{\in}\,[\theta,\theta+\varepsilon\ell),\\ u_k,\ t\in[\theta_k,\theta_k+\varepsilon\ell_k),\ k=\overline{1,s}, \end{cases}\\ v_\varepsilon(t)=v^0(t),\ t\in[t_1,t_2]. \end{cases} \tag{5}$$

Here, $\theta \in [t_0, t_1) = \theta + \varepsilon\ell$ is a continuity point from the right of the function $u^0(t)$, $\theta = \theta_1 < \theta_2 < ... < \theta_s + \varepsilon\ell_s$, $\varepsilon > 0$, $\ell > 0$, $\theta_k = \theta_{k-1} + \varepsilon\ell_{k-1}$, $\ell_k \geq 0$, $u_k \in U$, $k = \overline{1,s}$. s is an arbitrary natural number.

As it is seen, the control function $u_\varepsilon(t)$ depends on the choice of time θ, $\{\ell_k\}$, $\{u_k\}$.

Assume

$$h_1(\theta, u_k; t) = \lim_{\varepsilon \to 0} \frac{x_\varepsilon(t) - x^0(t)}{\varepsilon}, \tag{6}$$

$$h_2(\theta, u_k; t) = \lim_{\varepsilon \to 0} \frac{y_\varepsilon(t) - y^0(t)}{\varepsilon}. \tag{7}$$

Using (5), (6), (7), it is proved that the vector-functions $h_i(\theta, u_k; t)$, $i = 1, 2$ are the solutions of the following equations in variations

$$\dot{h}_1(\theta, u_k; t) = f_x(t, x^0(t), u^0(t)) h_1(\theta, u_k; t), \quad t > \theta,$$

$$\dot{h}_2(\theta, u_k; t) = g_y(t, y^0(t), v^0(t)) h_2(\theta, u_k; t), \quad t \in [t_1, t_2],$$

with initial conditions

$$h_1(\theta, u_k; \theta) = \sum_{k=1}^{s} \ell_k \left[f(\theta, x^0(\theta), u_k) - f(\theta, x^0(\theta), u^0(\theta)) \right], \quad h_1(\theta, u_k; t) = 0, \quad t < \theta,$$

$$h_2(\theta, u_k; t_1) = G_x(x^0(t_1)) h_1(\theta, u_k; t_1).$$

The collection θ, $\{\ell_k\}$, $\{u_k\}$ is said to be admissible with respect to $u_0(t)$, if the relations

$$\frac{\partial \Phi_k'(y^0(t_2))}{\partial y} h_2(\theta, u_k; t_2) = 0$$

are fulfilled for it.

Now, determine the "perturbed" control by the formula

$$\begin{cases} u_\mu(t) = u^0(t), \ t \in [t_0, t_1] \\ v_\mu(t) = \begin{cases} v^0(t), \ t \,\overline{\in}\, [\theta, \theta + \mu\rho), \\ v_k, \ t \in [\theta_k, \theta_k + \mu\rho_k), \ k = \overline{1,s}. \end{cases} \end{cases}$$

Here, θ is an arbitrary continuity point from the right of the control function $v^0(t)$, $\theta = \theta_1 < \theta_2 < ... < \theta_s + \mu\rho_s = \theta + \rho\mu$, $\mu > 0$, $\rho > 0$, $\theta_k = \theta_{k-1} + \mu\rho_{k-1}$, $\rho_k \geq 0$, $v_k \in V$, $k = \overline{1,s}$, s is an arbitrary natural number.

Let by the definition

$$q_1(\theta, v_k; t) = \lim_{\mu \to 0} \frac{x_\mu(t) - x^0(t)}{\mu},$$

$$q_2(\theta, v_k; t) = \lim_{\mu \to 0} \frac{y_\mu(t) - y^0(t)}{\mu}.$$

We can prove that $q_1(\theta, v_k; t) = 0$, $t \in [t_0, t_1]$, and $q_2(\theta, v_k; t)$ is a solution of the equation in variations

$$\dot{q}_2(\theta, v_k; t) = g_y(t, y^0(t), v^0(t)) q_2(\theta, v_k; t), \quad t \geq \theta,$$

with initial condition

$$q_2(\theta, v_k;\theta) = \sum_{k=1}^{s} \rho_k \left[g(\theta, y^0(\theta), v_k) - g(\theta, y^0(\theta), v^0(\theta)) \right], \quad q_2(\theta, v_k; t) = 0, \quad t < \theta .$$

The collection θ, $\{\rho_k\}$, $\{v_k\}$ is said to be admissible with respect the control function $v^0(t)$ if the relation

$$\frac{\partial \Phi'_k(y^0(t_2))}{\partial y} q_2(\theta, v_k; t_2) = 0$$

is fulfilled for it.

The following statement is true.

Theorem. In order the control $(u^0(t), v^0(t))$ be optimal in problem (1)-(5) it is necessary that the following relations be fulfilled.

1) $$\max_{z \in R(u^0, v^0)} \frac{\partial \varphi'(y^0(t_2), z)}{\partial y} h_2(\theta, v_k; t_2) \geq 0 ,$$

for all admissible with respect to $u^0(t)$ collections θ, $\{\ell_k\}$, $\{u_k\}$.

2) $$\max_{z \in R(u^0, v^0)} \frac{\partial \varphi'(y^0(t_2), z)}{\partial y} q_2(\theta, v_k; t_2) \geq 0 ,$$

for all admissible with respect to $v^0(t)$ collections θ, $\{\rho_k\}$, $\{v_k\}$.

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