LAPLACE TRANSFORMATION OF ERGODIC DISTRIBUTION OF THE STEP PROCESS OF SEMI-MARKOV RANDOM WALK WITH DELAYING SCREEN AT POSITIVE POINT

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In this paper the sequences of independent equiv distributed pair random variables is investigated and constructed the step process of semi-markovian random walk with delaying screen in $a > 0$. For obtained processes the evident form of Laplace transformation by time, the Fourier-Stieltjes transformation by phase of conditional distribution and unconditional distribution and evident form of Fourier-Stieltjes transformation by ergodic distribution was found.

It is well known that to investigation of ergodic distribution of step processes of semi-markovian random walk with delaying screen and without screen is devoted the sufficient papers. The main ergodic theorem for semi-markovian processes was proved by Smit. The general ergodic theorem was proved by Skorokhod. In the work by Ejov and Shurenkov is demonstated the easy proof of this theorem . In the Borovkov's work the ergodic theorem for processes of semi-markovian random walk with delaying screen. In the work by Skorokhod and Nasirova the ergodic theorem for complex processes of semi-markovian random walk with delaying screen in zero was proved.

In presented work in the case when delaying is in the classes of complex Laplace distribution the evident form of Laplace transformation by time, the Fourier-Stieltjes transformation by phase of conditional distribution and unconditional distribution and evident form of Fourier-Stieltjes transformation by ergodic distribution was found.

The main goal this report is formulated the evident form of Laplace transformation by time, Fourier-Stieltjes transformation by phase of conditional distribution and unconditional distribution evident form of Fourier-Stieltjes transformation by ergodic distribution $X(t,\omega)$.

Note that by Borovkov's [1] the evident form of ergodic distribution with random variables with delaying screen in zero was found, when the delaying is by arbitrary distribution.

But in presented report by other method for $X(t,\omega)$-processes [2] the evident form of Laplace transformation by time, the Fourier-Stieltjes transformation by phase of conditional distribution of step processes of semi-markovian random walk with delaying screen in point $a\ (a > 0)$ - $\tilde{\tilde{R}}_a(s,\beta / z)$. Also, the evident form of Fourier-Stieltjes transformation by phase of unconditional distribution of step processes of semi-markovian random walk with delaying screen in point $a\ (a > 0)$ - $\tilde{\tilde{R}}_a(s,\beta)$ and the evident form of Fourier-Stieltjes transformation by phase of ergodic distribution of step processes of semi-markovian random walk with delaying screen in point $a\ (a > 0)$ - $\tilde{R}_a(\beta)$ are found. Note that in this case the delaying is by complex Laplace distribution of order $(n,1)$ and presented method is applicable even in he case of complex Laplace distribution of order (n,m) (in last case we have the some technical difficults).

It is obvious that

$$\tilde{\tilde{R}}'_\beta(s,0/z) = -\int_{t=0}^{\infty} e^{-ist} E(X(t,\omega)/X(a,\omega)=z)\,dt\ ,$$

$$\tilde{\tilde{R}}'_\beta(s,a) = -\int_{t=0}^{\infty} e^{-ist} EX(t,\omega)\,dt$$

and therefore the Laplace transformation of higher moments of the process very important for calculus of mathematical expectation, dispersion of the higher moments of the process in any times.

Therefore first we make a integral equation for expressing $\tilde{\tilde{R}}_a(s,\beta/z)$, when the random delaying is via arbitrary distribution. We have

Theorem 1. The Laplace transformation by time, Fourier-Stieltjes transformation by phase of unconditional distribution of step processes of semi-markovian random walk with delaying screen in point a $(a>0)$ - $\tilde{\tilde{R}}_a(s,\beta)$ be satisfy the following integral equation

$$\begin{aligned}\tilde{\tilde{R}}_a(s,\beta/z) &= e^{i\beta z}\frac{1-\varphi(s)}{s} + \varphi(s)P\{\eta_1 < a-z\}\tilde{\tilde{R}}_a(s,x/a) + \\ &+ \varphi(s)\int_{y=-\infty}^{a}\tilde{\tilde{R}}_a(s,x/y)\,d_y P\{\eta_1 < y-z\}.\end{aligned} \tag{1}$$

The equation (1) for arbitrary distributed random variables $\xi_k(\omega), \eta_k(\omega)$, $k \geq 0$, can be solved by method of reccurent approximtion. Bu such solve not applicable for application. This equation have solve in evident form in the classes of complex Laplace distribution.

Theorem 2. Let

$$\eta_k(\omega) = \eta_{k1}^-(\omega) + \eta_{k2}^-(\omega) + ... + \eta_{kn}^-(\omega) - \eta_k^+(\omega), \quad k = \overline{1,\infty},$$

where $\eta_{ki}^-(\omega)$, $i=\overline{1,n}$, and $\eta_k^+(\omega)$ is the erlang distribution of first order with the parameters λ and μ responsibility and $E\eta_k(\omega) < 0$.

Then

$$\tilde{\tilde{R}}_a(s,\beta/z) = \frac{i\beta(\mu+i\beta)^n\mu^n\varphi(s)(1-\varphi(s))}{s\left[(\mu+k_1(s))^n - \mu^n\varphi(s)\right]\left[(i\beta-\alpha)(\mu+i\beta)^n + \mu^n\lambda\varphi(s)\right]}e^{k_1(s)z+i\beta a} +$$

$$+\frac{((i\beta-\alpha)(\mu+i\beta)^n[1-\varphi(s)]}{s\left[(i\beta-\alpha)(\mu+i\beta)^n + \mu^n\lambda\varphi(s)\right]}e^{i\beta z}\ .$$

The Laplace transformation by time, Fourier-Stieltjes transformation by phase of unconditional distribution of step processes of semi-markovian random walk with delaying screen in point a $(a>0)$ - $\tilde{\tilde{R}}_a(s,\beta)$ has the form:

$$\tilde{\tilde{R}}_a(s,\beta) = \frac{\beta(\mu+i\beta)^n(k_1(s)-\lambda)^2(1-\varphi(s))ie^{i\beta a}}{\lambda s k_1(s)\varphi(s)\left[(i\beta-\alpha)(\mu+i\beta)^n + \mu^n\lambda\varphi(s)\right]} +$$

$$+\frac{\mu^{n}(i\beta-\alpha)[1-\varphi(s)]}{s\left[(i\beta-\alpha)(\mu+i\beta)^{n}+\mu^{n}\lambda\varphi(s)\right]}.$$

We find the evident form of by the phase of ergodic distribution of step processes of semi-markovian random walk with delaying screen in point a $(a>0)$

$$\widetilde{R}_{a}(\beta)=\frac{(n\lambda-\mu)(\mu+i\beta)^{n}e^{i\beta a}}{\mu\left[\lambda\sum_{j=1}^{n}(\mu+i\beta)^{n-j}\mu^{j-1}-(\mu+i\beta)^{n}\right]}.$$

In particular, from getting distribution we found the first and second moments of ergodic distribution for investigated process as

$$EX_{a}(\omega)=-\widetilde{R}_{a}'(a)=\frac{-n(n+1)\lambda+2a\mu(\mu-n\lambda)}{2\mu(\mu-n\lambda)}, \quad \text{for} \quad \mu>n\lambda$$

and

$$DX_{a}(\omega)=\frac{n(n+1)\lambda\left[4(n+2)\mu-n(n+5)\lambda\right]}{12\mu^{2}(\mu-n\lambda)^{2}}, \qquad \text{for} \quad \mu>n\lambda.$$

References

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