# INVESTIGATION OF LOW FREQUENCY TUBE RESONATOR LUMPED INHOMOGENEITY FOR VIBRATION-FREQUENCY DENSITOMETER LIQUID MEDIA 

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Most currently known vibration-frequency densitometer liquid media as sensitive elements have a high-flow mechanical systems - resonators (operating frequency over 1000 Hz ) 1 /. This restricts the use of these devices clean fluids and prevents their effective use to determine the density of two-phase media type liquid - solid inclusion, because high natural frequency is the phase shift between the movement of liquid and solid particles. In this regard, to work with these fluids is recommended to use low-frequency resonators $/ 2 /$. However, reducing the frequency of natural oscillations due to the lower quality of the mechanical oscillation system, and hence the accuracy of measurement. Therefore, the search methods of reducing their frequency resonators VCHP with their acceptable quality of some interest.

The present work is devoted research of the tubular resonator with is concentrated th heterogeneity in the form of rigidly attached inertial weights. It is known that introduction in initial oscillatory system of additional weights leads to decrease in own frequency of system. On the other hand, the entered weights change the form to-lebany the resonator and by that influence size of jet forces in places zashchemle-nija the resonator - the basic source of dispersion of oscillatory energy, and consequently also good quality decrease. In this connection the research problem consists in search of such parametres of inertial weights and their sites which would provide is set th frequency of own fluctuations at the minimum size of jet efforts in places of a jamming of the resonator.

## I. Compiling and research of frequence equation.

Scheme of the investigated resonator shown in Fig. 1


Fig. 1 General scheme of the resonator
Given the symmetry of fluctuations in the tube about its mid-cavity design scheme can be represented as in Fig.2.


Fig. 2 Settlement scheme of the resonator

Half of the tube divided into two sections: the first from the place of incorporation to the inertial mass, the second of the inertial mass to the middle of the tube.
Let the length of sections $l_{1}$ and $l_{2}$, if so $l=l_{1}+l_{2}$. Based on previous assumptions and notations for the two plots equations of motion are written as

$$
\begin{equation*}
E I \frac{\partial^{4} y_{i}}{\partial x^{4}}+\left(m_{m}+m_{\nsim}\right) \frac{\partial^{2} y_{i}}{\partial t^{2}}=0, i=1,2, \ldots \tag{1}
\end{equation*}
$$

We believe that

$$
y(x, t)=v(t) z(x)
$$

where $z(x)$ is the solution of differential equations IY comment

$$
z^{I Y}-v^{4} z=0
$$

$v(t)$ is solution of differential equations of 2-order

$$
\begin{gathered}
\ddot{v}_{i}+\omega v_{i}=0 \\
\omega=v^{2} \sqrt{\frac{E I}{m_{m}+m_{\varkappa}}}
\end{gathered}
$$

Here $\omega=2 \pi f_{0}$ ( $f_{0}-$ own frequency of fluctuations of a tube) is circular frequency of fluctuations. The common decision of the equation (1) for both sites looks like

$$
z(x)=A S\left(v x_{i}\right)+B T\left(v x_{i}\right)+C U\left(v x_{i}\right)+D V\left(v x_{i}\right)
$$

For the first site from a jamming condition at $x_{i}=0$ is had $z_{1}(0)=z_{1}^{\prime}(0)=0$. From here $z_{1}\left(x_{1}\right)$ it is possible to present in a kind

$$
z_{1}\left(x_{1}\right)=a_{1} U\left(v l_{1}\right)+b_{1} V\left(v l_{1}\right)
$$

where $a_{1}, b_{1}$ are subject to definition from boundary conditions of interface.
For the second site from a condition of absence of an angle of rotation and cutting forces at $x_{2}=0$ it is had, $z_{2}^{\prime}\left(x_{2}\right)=z_{2}^{\prime \prime \prime}(0)=0$ i.e.

$$
z_{2}\left(x_{2}\right)=a_{2} S\left(v l_{2}\right)+b_{2} U\left(v l_{2}\right)
$$

where $a_{2}, b_{2}$ are subject to definition from boundary conditions of interface.
In a point of interface of sites at $x_{1}=l_{1}$ also $x_{2}=-l_{2}$ it is had

$$
\begin{aligned}
& z_{1}\left(l_{1}\right)=z_{2}\left(-l_{2}\right) \\
& z_{1}^{\prime}\left(l_{1}\right)=z_{2}^{\prime}\left(-l_{2}\right) \\
& z_{2}^{\prime \prime}\left(-l_{2}\right)=z_{1}^{\prime \prime}\left(l_{1}\right)+M / E I \\
& z_{2}^{\prime \prime \prime}\left(-l_{2}\right)=z_{1}^{\prime \prime \prime}\left(l_{1}\right)+F / E I
\end{aligned}
$$

Where $M$ and $F$ - the moment and the force operating from outside of a tube on weight. According to first two conditions (3) it is had

$$
\begin{align*}
& a_{1} U\left(v l_{1}\right)+b_{1} V\left(v l_{1}\right)=a_{2} S\left(v l_{2}\right)+b_{2} U\left(v l_{2}\right)  \tag{3}\\
& a_{1} T\left(l_{1}\right)+b_{1} U\left(v l_{1}\right)=-a_{2} V\left(v l_{2}\right)-b_{2} T\left(v l_{2}\right)
\end{align*}
$$

We will enter following matrixes

$$
A_{1}=\binom{a_{1}}{b_{1}} ; A_{2}=\binom{a_{2}}{b_{2}} ; B_{1}=\left(\begin{array}{cc}
S\left(v l_{2}\right) & U\left(v l_{2}\right)  \tag{4}\\
-V\left(v l_{2}\right) & -T\left(v l_{2}\right)
\end{array}\right) ; B_{2}=\left(\begin{array}{cc}
U\left(v l_{1}\right) & V\left(v l_{1}\right) \\
T\left(v l_{1}\right) & U\left(v l_{1}\right)
\end{array}\right) .
$$

Then the system of the equations (4) can be written down in a following kind

$$
B_{2} A_{1}=B_{1} A_{2}
$$

From here

$$
\begin{equation*}
A_{2}=B_{1}^{-1} B_{2} A_{1} . \tag{5}
\end{equation*}
$$

From the third and fourth conditions (3) it is had we

$$
\begin{align*}
& a_{1} S\left(v l_{1}\right)+b_{1} T\left(v_{1}\right)+\frac{M}{v^{2} v E I}=a_{2} U\left(v l_{2}\right)+b_{2} S\left(v l_{2}\right)  \tag{6}\\
& a_{1} V\left(v_{1}\right)+b_{1} T\left(v_{1}\right)+\frac{F}{v^{3} v E I}=-a_{2} T\left(v l_{2}\right)-b_{2} V\left(v_{2}\right) .
\end{align*}
$$

Will enter matrixes:

$$
B_{3}=\left(\begin{array}{cc}
U\left(l_{2}\right) & S\left(l_{2}\right) \\
-T\left(v l_{2}\right) & -V\left(v_{2}\right.
\end{array}\right) ; B_{4}=\left(\begin{array}{ll}
S\left(l_{1}\right) & T\left(w_{1}\right) \\
V\left(v_{1}\right) & S\left(l_{1}\right)
\end{array}\right) ; C=\binom{\frac{M}{v v^{2} E I}}{\frac{F}{v v^{3} E I}} .
$$

Then the equation (5) we will write down in a kind

$$
\begin{equation*}
B_{4} A_{1}+C=B_{3} A_{2} \tag{7}
\end{equation*}
$$

We will find $M$ and $F$. From the equation of a rotary motion of weight it is had

$$
\begin{equation*}
I_{c} \ddot{\theta}=M \tag{8}
\end{equation*}
$$

where $\theta=z_{1}\left(l_{1}\right) v(t)$ is a tube angle of rotation in a place of fastening of weight; $I_{c}$ is the moment of inertia of weight.
From a parity (7) follows

$$
\begin{equation*}
M=-2 v E I v^{2} \alpha\left(T\left(v l_{1}\right) U\left(v_{1}\right)\right) A_{1}, \tag{9}
\end{equation*}
$$

where $\alpha=\frac{I_{c} \nu^{3}}{m_{c}}$
From the equation of progress of weight $m_{c}$ it is found (10)

$$
\begin{equation*}
m_{c} \ddot{y}_{1}=-F, \tag{10}
\end{equation*}
$$

where $y_{1}$-linear moving of weight $m_{c}$.
From expression (10)

$$
\begin{equation*}
F=2 v E I v^{3} \beta\left(U\left(v l_{1}\right) V\left(v l_{1}\right)\right) A_{1} \tag{11}
\end{equation*}
$$

it is received (11) where $\beta=\frac{m_{c}}{2 m} v$, and $m=m_{m}+m_{\leadsto c}$.
Taking into account expressions (9) and (11) for $C$ it is had

$$
\begin{equation*}
C=2 D A_{1} . \tag{12}
\end{equation*}
$$

Here

$$
D=\left(\begin{array}{cc}
-\alpha T\left(v l_{1}\right) & -\alpha U\left(o l_{1}\right) \\
\beta U\left(v l_{1}\right) & \beta V\left(v_{1}\right)
\end{array}\right) .
$$

Having substituted in (6) expressions (5) and (12), we will receive

$$
\begin{equation*}
X A_{1}=0 \tag{13}
\end{equation*}
$$

where $X=B_{3} B_{1}^{-1} B_{2}-B_{4}-2 D$, and elements

$$
\begin{aligned}
& x_{11}=\frac{1}{2}\left(\cos v l_{2} \operatorname{sh} v l+\operatorname{ch} v l_{2} \sin v l\right)-\alpha\left(\operatorname{sh} v l_{1}+\sin v l_{1}\right) * \\
& *\left(\cos v l_{2} \sin v l_{2}+\operatorname{sh} v l_{2} \cos v l_{2}\right) \\
& x_{12}=\frac{1}{2}\left(\cos v l_{2} \operatorname{ch} v l-\operatorname{ch} v l_{2} \cos v l\right)+\alpha\left(\cos v l_{1}-\operatorname{ch} v l_{1}\right) * \\
& *\left(\operatorname{ch} v l_{2} \sin v l_{2}+\operatorname{sh} v l_{2} \cos v l_{2}\right) \\
& x_{21}=\frac{1}{2}\left(\sin v l_{2} \operatorname{sh} v l-\operatorname{sh} v l_{2} \sin v l\right)-\beta\left(\cos v l_{1}-\operatorname{ch} v l_{1}\right) * \\
& *\left(\operatorname{ch} v l_{2} \sin v l_{2}+\operatorname{sh} v l_{2} \cos v l_{2}\right) \\
& x_{22}=\frac{1}{2}\left(\sin v l_{2} \operatorname{ch} v l+\operatorname{sh} v l_{2} \cos v l\right)-\beta\left(\sin v l_{1}-\operatorname{sh} v l_{1}\right) * \\
& *\left(\operatorname{ch} v l_{2} \sin v l_{2}+\operatorname{sh} v l_{2} \cos v l_{2}\right) .
\end{aligned}
$$

The decision of the equation distinct from zero (13) is from $\operatorname{det} X=0$ condition performance. Opening the given determinant, we will receive the equation for parametre $v$.

$$
\begin{gather*}
A(v l / 2)-2 A\left(v l_{2}\right) D\left(v l_{1}\right) \alpha \beta-\left(A\left(v l_{1}\right) A\left(v l_{2}\right)+D\left(v l_{1}\right) S\left(v l_{2}\right)\right) \alpha+ \\
+\left(D\left(v l_{1}\right) C\left(v l_{2}\right)-B\left(v l_{1}\right) A\left(v l_{2}\right)\right) \beta=0 \tag{14}
\end{gather*}
$$

Here $A(v x), B(v x), C(v x), D(v x)$ and $S(v x)$ so-called functions of Pragera and
Gogenemzera / 3/:

$$
\begin{aligned}
& A(v x)=\operatorname{ch} v x \sin v x+\operatorname{sh} v x \cos v x \\
& B(v x)=\text { ch } v x \sin v x-\operatorname{sh} v x \cos v x \\
& C(v x)=2 \operatorname{ch} v x \cos v x \\
& D(v x)=2 \operatorname{sh} v x \sin v x
\end{aligned}
$$

Let's check up the received results about the coordination with known settlement parities. For example, in the absence of weights $m_{c}$ factors $\alpha$ also $\beta$ become equal to zero and the equation (14) becomes

$$
\begin{equation*}
\operatorname{ch} \frac{v l}{2} \sin \frac{v l}{2}+\operatorname{sh} \frac{v l}{2} \cos \frac{v l}{2}=0 \tag{15}
\end{equation*}
$$

Nonzero decision (15) for symmetric fluctuations $v=\frac{4.73}{l}$ - i.e. is received the known decision of the equation of own fluctuations of a tube with the jammed ends.
II. Detection of condition minimization of reactive forces in fixatien places of resomator We will pass to the second investigation phase, namely a finding of analytical expression for definition of size of jet forces in places of a jamming of a tube. Force in the closing:

$$
F=E I y_{1}^{\prime \prime \prime}(0, t)
$$

Peak value of this size:

$$
\begin{equation*}
F_{A}=E I z_{1}^{\prime \prime \prime}(0) \tag{16}
\end{equation*}
$$

Taking into account (2) it is had

$$
z_{1}\left(x_{1}\right)=\left(U\left(v l_{1}\right) V\left(v l_{1}\right)\right) A_{1}
$$

whence

$$
z_{1}^{\prime \prime \prime}(0)=v^{3}(0,1) A_{1}
$$

According to (13) $A_{1}$ being own vector of the matrix $X$ calculated at value $v$, being the decision of the equation of frequencies (14), it is defined with accuracy scalar multiplier

$$
A_{1}=\frac{\lambda}{E I}\binom{x_{12}}{-x_{11}}
$$

Then for $F_{A}$ it is had

$$
\begin{equation*}
F_{A}=\lambda v^{3} x_{11} \tag{17}
\end{equation*}
$$

where $\lambda$ is a constant defining intensity of fluctuations, i.e. depending on entry conditions. As intensity of fluctuations the amplitude of movings of an average point of a tube can be accepted $h$. We will believe that irrespective of system parametres in it the fixed amplitude is provided $h$. We will express it through these parametres. As,

$$
h=z(l / 2)
$$

that we have

$$
h=\frac{\left.\lambda\left(V\left(v l_{2}\right) V\left(v l_{1}\right)+T\left(v l_{2}\right) U\left(v l_{1}\right)\right) x_{11}-\left(V\left(v l_{2}\right) U\left(v l_{1}\right)+T\left(v l_{2}\right) T\left(v l_{1}\right)\right) x_{12}\right)}{E I\left(V\left(v l_{2}\right) U\left(v l_{2}\right)-S\left(v l_{2}\right) T\left(v l_{2}\right)\right)}
$$

considering value $\lambda$ from (17) in the formula (18) it is had

$$
F_{A}=E I d^{4}\left(n^{4}-1\right)\left[\frac{\lambda_{i}}{l}\right]^{3} \frac{h\left(V\left(v l_{2}\right) U\left(v l_{2}\right)-S\left(v l_{2}\right) T\left(v l_{2}\right)\right) x_{11}}{\left(V\left(v l_{2}\right) V\left(v l_{1}\right)+T\left(v l_{2}\right) U\left(v l_{1}\right)\right) x_{11}-\left(V\left(v l_{2}\right) U\left(v l_{1}\right)+T\left(v l_{2}\right) T\left(v l_{1}\right)\right) x_{12}}
$$

From this it follows that the size of cross-section reactions $F_{A}$ equals to zero at performance of a following condition

$$
V\left(v l_{2}\right) U\left(v l_{2}\right)-S\left(v l_{2}\right) T\left(v l_{2}\right)=0
$$

the received results testify to existence of infinite set of parametres of the offered resonator at which equality to zero of jet forces in places of a jamming of a tube is provided.

## Conclusion

Experience of working out vibrating densitymeters shows that the tube material, its internal diameter and a thickness get out proceeding from technological conditions of measurement, and own frequency of fluctuations of a condition of maintenance of the set absolute sensitivity. Then selecting parametres of the concentrated weights proceeding from constructive reasons other parametres can be defined as follows

- Definition of value $v$ of parametre proceeding from set own frequency of fluctuations of the resonator;
- Definition of length $l_{2}$ of a site from the equation (19);
- Definition of length $l_{1}$ of a site from the equation (14)

At drawing up of the equation of movement (1) us has not been considered taking place in real oscillatory systems a friction. In this connection in practice of full decrease in jet forces in places of a jamming of a tube of the resonator it will not be possible. However the qualitative party of a problem remains without changes.

## References

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