CALCULATION OF SECTORAL REGIONAL DEVELOPMENT INDEXES (on an example of education service support in regions)

Ramin Rzayev[1], Elchin Aliyev[2]

Institute of Cybernetics of ANAS, Baku, Azerbaijan
[1]*raminrza@yahoo.com*, [2]*elchin@sinam.net*

The state selective support of regions frequently occurs in conditions of incompleteness, fuzziness and/or inconsistency of the available information [1]. It takes a key place in the national policy on region development which because of absence science-based and information-analytical (dataware) support of regional goal-oriented program is frequently carried out in uncertainty conditions. There is the most complicated problem of objective selection of the neediest regions for which one can perceptibly and justifiably to spend the state resources.

Feature of management problems in uncertainty conditions consists that inputs and outputs measurements (supervision) are carried out at a level of «soft computing» which adequate representation is possible due to their representation in the type of fuzzy sets [2, 3]. In particular, the incoming data from the respondents characterizing different spheres of the region socio-economic structure (for example, such as a demography, nonmaterial, manpower resources and employment, social maintenance, industry, agriculture, education, public services, construction activity, medical care, etc.), is described by weakly structured and/or unstructured data that is data about whom their accessory to the certain type is known only. Therefore for the estimation of regions socio-economic development levels on different sectors it is offered to use fuzzy (verbal) models based on implicative form «If-then». The given approach allows to process weakly structured data and to involve in computational process various qualitative categories. In particular, the education sphere in the regions characterized by respondents' weakly structured data is considered. On the basis of these data sectoral indexes of development are calculated and regions are ranked.

For estimation of development levels of the region education systems we shall take advantage of a method of not precise conclusion [4]. For this purpose let us take an advantage of sufficient in our opinion of number of implicative rules constructed on the basis of linguistic variables from table 1:

1. If $x_1 = \tilde{X}_1$ and $x_2 = \tilde{X}_2$ and $x_5 = \tilde{X}_5$ and $x_6 = \tilde{X}_6$ and $x_9 = \tilde{X}_9$ and $x_{10} = \tilde{X}_{10}$, then $Y = \tilde{S}$.
2. If $x_1 = \neg\tilde{X}_1$ and $x_2 = \tilde{X}_2$ and $x_5 = \tilde{X}_5$ and $x_6 = \tilde{X}_6$ and $x_9 = \tilde{X}_9$ and $x_{10} = \tilde{X}_{10}$ and $x_7 = \tilde{X}_7$ and $x_{11} = \tilde{X}_{11}$, then $Y = \tilde{S}$.
3. If $x_1 = \tilde{X}_1$ and $x_2 = \tilde{X}_2$ and $x_3 = \tilde{X}_3$ and $x_4 = \tilde{X}_4$ and $x_5 = \tilde{X}_5$ and $x_6 = \tilde{X}_6$ and $x_9 = \tilde{X}_9$ and $x_{10} = \tilde{X}_{10}$, then $Y = M\tilde{S}$.
4. If $x_1 = \tilde{X}_1$ and $x_2 = \tilde{X}_2$ and $x_3 = \tilde{X}_3$ and $x_4 = \tilde{X}_4$ and $x_5 = \tilde{X}_5$ and $x_6 = \tilde{X}_6$ and $x_7 = \tilde{X}_7$ and $x_9 = \tilde{X}_9$ and $x_{10} = \tilde{X}_{10}$, then $Y = V\tilde{S}$.
5. If $x_1 = \tilde{X}_1$ and $x_2 = \tilde{X}_2$ and $x_3 = \tilde{X}_3$ and $x_4 = \tilde{X}_4$ and $x_5 = \tilde{X}_5$ and $x_6 = \tilde{X}_6$ and $x_7 = \tilde{X}_7$ and $x_8 = \tilde{X}_8$ and $x_9 = \tilde{X}_9$ and $x_{10} = \tilde{X}_{10}$ and $x_{11} = \tilde{X}_{11}$ and $x_{12} = \tilde{X}_{12}$, then $Y = \tilde{P}$.
6. If $x_1 = \neg\tilde{X}_1$ and $x_2 = \neg\tilde{X}_2$ and $x_5 = \neg\tilde{X}_5$ and $x_6 = \neg\tilde{X}_6$ and $x_9 = \tilde{X}_9$ and $x_{10} = \neg\tilde{X}_{10}$, then $Y = U\tilde{S}$.

Table 1

Linguistic parameters of the education system in fuzzy infomedia

Symb. not.	The linguistic variables accepted as parameters	Qualitative criterion of an estimation (the fuzzy term-set)	Symb. not.
x_1	Education spendings	Full	$\tilde{X}_1$
x_2	Seating accommodation at preschool institutions	Enough	$\tilde{X}_2$
x_3	Seating accommodation at boarding school	Enough	$\tilde{X}_3$
x_4	Seating accommodation at schools for handicapped children	Enough	$\tilde{X}_4$
x_5	Seating accommodation at secondary school	Enough	$\tilde{X}_5$
x_6	Number of teachers	Enough	$\tilde{X}_6$
x_7	Modern training equipment supply	Enough	$\tilde{X}_7$
x_8	Computer supply	Enough	$\tilde{X}_8$
x_9	Number of the educational institutions, demanding thorough repair	One ore several schools	$\tilde{X}_9$
x_{10}	The population aggregate which are not having the educational institutions	Be absent	$\tilde{X}_{10}$
x_{11}	Specialized secondary schools	Enough number	$\tilde{X}_{11}$
x_{12}	Seating accommodation at the new schools up built at the expense of government	Enough	$\tilde{X}_{12}$

For values (terms) of linguistic variable Y used in the rules on the basis of discrete set $I = \{0{,}1; 0{,}2; ...; 1\}$ let us construct corresponding fuzzy sets by the instrumentality of following membership functions: $\tilde{S}$ =satisfactory – $\mu_{\tilde{S}}(x) = x,\ x \in I$; $M\tilde{S}$ =more than satisfactory – $\mu_{M\tilde{S}}(x) = \sqrt{x},\ x \in I$; $\tilde{P}$ =high – $\mu_{\tilde{P}}(x) = \begin{cases} 1, x = 1, \\ 0, x < 1; \end{cases}$ $V\tilde{S}$ =very satisfactory – $\mu_{V\tilde{S}}(x) = x^2,\ x \in I$; $U\tilde{S}$ =unsatisfactory – $\mu_{U\tilde{S}}(x) = 1 - x,\ x \in I$. Terms of input variables used in rules are designated by fuzzy sets $\tilde{X}_k$ $(k = \overline{1{,}12})$ with gauss membership functions $\mu_{\tilde{X}_k}(u) = e^{-\frac{(u-u_0)^2}{\sigma^2}}$, where u_0 is the center; σ^2 is the density of distribution of regional data in the define statistical interval. Then on the basis of 5 arbitrary regions described by the statistical data (tab. 1) let us generate the estimation criteria of the education system:

$$\tilde{X}_1 = \frac{0.3}{u_1} + \frac{0.2}{u_2} + \frac{0.6}{u_3} + \frac{1}{u_4} + \frac{0.8}{u_5};\ \tilde{X}_2 = \frac{0.7}{u_1} + \frac{0.4}{u_2} + \frac{0.55}{u_3} + \frac{1}{u_4} + \frac{0.9}{u_5};$$

$$\tilde{X}_3 = \frac{0.4}{u_1} + \frac{0.3}{u_2} + \frac{0.7}{u_3} + \frac{0.5}{u_4} + \frac{0.95}{u_5};\ \tilde{X}_4 = \frac{0.95}{u_1} + \frac{0.25}{u_2} + \frac{0.8}{u_3} + \frac{0.4}{u_4} + \frac{0.7}{u_5};$$

$$\tilde{X}_5 = \frac{0.7}{u_1} + \frac{0.35}{u_2} + \frac{1}{u_3} + \frac{0.9}{u_4} + \frac{0.25}{u_5};\ \tilde{X}_6 = \frac{0.1}{u_1} + \frac{0.2}{u_2} + \frac{0.9}{u_3} + \frac{0.4}{u_4} + \frac{0.7}{u_5};$$

$$\tilde{X}_7 = \frac{0.25}{u_1} + \frac{0.3}{u_2} + \frac{0.2}{u_3} + \frac{0.1}{u_4} + \frac{0.5}{u_5};\ \tilde{X}_8 = \frac{0.5}{u_1} + \frac{0.75}{u_2} + \frac{0.9}{u_3} + \frac{0.4}{u_4} + \frac{1}{u_5};$$

$$\tilde{X}_9 = \frac{0.8}{u_1} + \frac{0.65}{u_2} + \frac{0.15}{u_3} + \frac{0.95}{u_4} + \frac{0.4}{u_5};\ \tilde{X}_{10} = \frac{0.95}{u_1} + \frac{0.15}{u_2} + \frac{0.8}{u_3} + \frac{0.3}{u_4} + \frac{0.6}{u_5};$$

$$\tilde{X}_{11} = \frac{0.15}{u_1} + \frac{0.9}{u_2} + \frac{1}{u_3} + \frac{0.45}{u_4} + \frac{0.8}{u_5};\ \tilde{X}_{12} = \frac{0.5}{u_1} + \frac{0.2}{u_2} + \frac{0.8}{u_3} + \frac{0.3}{u_4} + \frac{0.95}{u_5}.$$

Further, by a minimum principle for the left parts of fuzzy rules we shall define corresponding fuzzy sets $\tilde{M}_i$ $(i=\overline{1,6})$. Then rules can be presented in following more compact appearance:

- if $x=\tilde{M}_1$, then $Y=\tilde{S}$, where $\tilde{M}_1=\left\{\frac{0.1}{u_1};\frac{0.15}{u_2};\frac{0.15}{u_3};\frac{0.3}{u_4};\frac{0.25}{u_5}\right\}$;
- if $x=\tilde{M}_2$, then $Y=\tilde{S}$; where $\tilde{M}_2=\left\{\frac{0.1}{u_1};\frac{0.15}{u_2};\frac{0.15}{u_3};\frac{0.1}{u_4};\frac{0.25}{u_5}\right\}$;
- if $x=\tilde{M}_3$, then $Y=M\tilde{S}$, where $\tilde{M}_3=\left\{\frac{0.1}{u_1};\frac{0.15}{u_2};\frac{0.15}{u_3};\frac{0.3}{u_4};\frac{0.25}{u_5}\right\}$;
- if $x=\tilde{M}_4$, then $Y=V\tilde{S}$, where $\tilde{M}_4=\left\{\frac{0.1}{u_1};\frac{0.15}{u_2};\frac{0.15}{u_3};\frac{0.1}{u_4};\frac{0.25}{u_5}\right\}$;
- if $x=\tilde{M}_5$, then $Y=\tilde{P}$, where $\tilde{M}_5=\left\{\frac{0.1}{u_1};\frac{0.15}{u_2};\frac{0.15}{u_3};\frac{0.1}{u_4};\frac{0.25}{u_5}\right\}$;
- if $x=\tilde{M}_6$, then $Y=U\tilde{S}$, where $\tilde{M}_6=\left\{\frac{0.05}{u_1};\frac{0.6}{u_2};\frac{0}{u_3};\frac{0}{u_4};\frac{0.1}{u_5}\right\}$.

For transformation of these rules we shall take advantage of Lukasevich's implication operation: $\mu_{\tilde{D}}(u,j)=\min(1,1-\mu_{\tilde{M}}(u)+\mu_{\tilde{Y}}(j))$. As a result for each pair $(u,j)\in U\times J$ on product $U\times J$ one can obtaine fuzzy relations $\tilde{D}_i$ $(i=\overline{1,6})$ in following matrix types:

$\tilde{D}_1=$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	0.9	1	1	1	1	1	1	1	1	1	1
u_2	0.85	0.95	1	1	1	1	1	1	1	1	1
u_3	0.85	0.95	1	1	1	1	1	1	1	1	1
u_4	0.7	0.8	0.9	1	1	1	1	1	1	1	1
u_5	0.75	0.85	0.95	1	1	1	1	1	1	1	1

$\tilde{D}_2=$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	0.9	1	1	1	1	1	1	1	1	1	1
u_2	0.85	0.95	1	1	1	1	1	1	1	1	1
u_3	0.85	0.95	1	1	1	1	1	1	1	1	1
u_4	0.9	1	1	1	1	1	1	1	1	1	1
u_5	0.75	0.85	0.95	1	1	1	1	1	1	1	1

$\tilde{D}_3=$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	0.9	1	1	1	1	1	1	1	1	1	1
u_2	0.85	1	1	1	1	1	1	1	1	1	1
u_3	0.85	1	1	1	1	1	1	1	1	1	1
u_4	0.7	1	1	1	1	1	1	1	1	1	1
u_5	0.75	1	1	1	1	1	1	1	1	1	1

$\tilde{D}_4=$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	0.9	0.91	0.94	0.99	1	1	1	1	1	1	1
u_2	0.85	0.86	0.89	0.94	1	1	1	1	1	1	1
u_3	0.85	0.86	0.89	0.94	1	1	1	1	1	1	1
u_4	0.9	0.91	0.94	0.99	1	1	1	1	1	1	1
u_5	0.75	0.76	0.79	0.84	0.91	1	1	1	1	1	1

$\tilde{D}_5 =$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1
u_2	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	1
u_3	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	1
u_4	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1
u_5	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	1

$\tilde{D}_6 =$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	1	1	1	1	1	1	1	1	1	1	0.95
u_2	1	1	1	1	1	0.9	0.8	0.7	0.6	0.5	0.4
u_3	1	1	1	1	1	1	1	1	1	1	1
u_4	1	1	1	1	1	1	1	1	1	1	1
u_5	1	1	1	1	1	1	1	1	1	1	0.9

Further, on the basis of intersection these relations one can obtain the general required decision:

$\tilde{D} =$

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
u_1	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.95
u_2	0.85	0.85	0.85	0.85	0.85	0.85	0.8	0.7	0.6	0.5	0.4
u_3	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	1
u_4	0.7	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	1
u_5	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.9

At last, fuzzy sets obtained for regions let us compare on the interval I like that.

For first region (u_1) from last matrix it is had:

$$E_1 = \left\{ \frac{0.9}{0} + \frac{0.9}{0.1} + \frac{0.9}{0.2} + \frac{0.9}{0.3} + \frac{0.9}{0.4} + \frac{0.9}{0.5} + \frac{0.9}{0.6} + \frac{0.9}{0.7} + \frac{0.9}{0.8} + \frac{0.9}{0.9} + \frac{0.95}{1.0} \right\}.$$

On the basis of it let us construct level sets $E_{j\alpha}$ and calculate corresponding cardinal number by formula $M(E_{j\alpha}) = \sum_{j=1}^{n} \frac{x_j}{n}$:

- $0 < \alpha < 0.9$: $\Delta\alpha = 0.9$; $E_{1\alpha} = \{0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1\}$; $M(E_{1\alpha}) = 0{,}5$;
- $0.9 < \alpha < 0.95$: $\Delta\alpha = 0.05$; $E_{1\alpha} = \{1\}$; $M(E_{1\alpha}) = 1$.

Then a numerical estimation for E_1 let us obtain like that:

$$F(E_1) = \frac{1}{\alpha_{\max}} \int_0^{\alpha_{\max}} M(E_{1\alpha}) d\alpha = \frac{1}{0.95} \int_0^{0.95} M(E_{1\alpha}) d\alpha = \frac{1}{0.95} (0.9 \cdot 0.5 + 0.05 \cdot 1) = 0.526 .$$

Similarly we obtain numerical estimations for other regions: $F(E_2) = 0.426$; $F(E_3) = 0.575$; $F(E_4) = 0.565$; $F(E_5) = 0.583$. Region having the highest sectoral index of education organization is considered as the best. In our case it will be fifth region, and further descending ordering: the third, the fourth, the first and the second.

Literature

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