ON THE OPTIMAL CONTROL PROBLEM FOR PROCESSES REPRESENTED MULTIPARAMETRIC SEQUENTIAL MACHINERIES

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Many problems of science and technics are shown with multiparametric finite linear difference equation systems.

Such problems are used in radiotechnics, telemetrics, automatic controlling, and cosmic researching. Linear sequential machines which frequently used in constructing modern calculating systems, imitative modeling of object and processes are described with such systems. Therefore this systems and the problems which are given by this systems are demanded to analyse. It is necessary and actual to find the methods for solving an optimal control problems.

In this study multiparametric dual linear difference equation systems are analysed and we research an optimal controlling problems which are shown below for this systems:

$$\xi_\nu s(c) = \Phi_\nu(c)s(c) \oplus \Psi_\nu(c)x(c),\ c \in G_d \qquad \text{v= 1,...,k,} [GF(2)]$$

$$s(c^0) = s^0,$$

$$x(c) \in \hat{X}, c \in \hat{G}_d,$$

$$J(x) = a's(c^L) \to \min$$

where $c = (c_1,...,c_k) \in G_d = \{c \mid c \in Z^k, c_1^0 \le c_1 \le c_1^{L_1},...,c_k^0 \le c_k \le c_k^{L_k}, c_i \in Z\}$ are the points in Z^k. L_i, i=1,…,k where k is an integer, $Z = \{...,-1,0,1,...\}$, set of integers, $s(c) \in S, x(c) \in X$; $S = [GF(2)]^m$, $X = [GF(2)]^r$ are state and entrance index (alphabet) respectively. where $s(c)$ and $x(c)$ are defined over the set Z^K as an m and r dimensional state and entrance vectors.

$\xi_\nu s(c) = s(c + e_\nu); e_\nu = (0,...,0,\overset{\nu}{1},0,...,0)$, ν=1,…,k is a shift operator which is described below as $\{\Phi_\nu(c), \nu = 1,...k\}$, $\{\Psi_\nu, \nu = 1,...,k\}$ are characteristic booolean translation matries of size $m \times n$ and $m \times r$ respectively. $[GF(2)]$ Galois Field, $s(c^0) = s^0$ is the initial state vector, $a = (a_1, a_2,..., a_m)$ boolean vector, the prime on a is the transapose, $L = L_1 + L_2 + ... + L_k$ is the period of this process. $\hat{G}_d = G_d \setminus \{c^L\}$ and $\hat{X} = \{x(c), c \in \hat{G}_d\}$.

In this study theorems on necessary and sufficient conditions are proved for shown problems. Moreover the different necessary and sufficient conditions theorems for stable systems are also given also.

I. OPTIMAL CONTROL PROBLEM IN MULTI PARAMETER LINEAR DIFFERENCE EQUATION SYSTEM OF GIVEN PROCESSES

Multi parameter difference equation system is defined as follows[3]:

$$\xi_\nu s(c) = \Phi_\nu(c)s(c) \oplus \Psi_\nu(c)x(c) \qquad \nu\text{= 1,...,}k, [GF(2)] \tag{1.1}$$

$$s(c^0) = s^0$$

where $c = (c_1,...,c_k) \in G_d = \{c \mid c \in Z^k, c_1^0 \le c_1 \le c_1^{L_1},...,c_k^0 \le c_k \le c_k^{L_k}, c_i \in Z\}$ are the points in Z^k, L_i, i=1,…,k where k is an integer, $Z = \{...,-1,0,1,...\}$, set of integers, for $s(c) \in S, x(c) \in X$; $S = [GF(2)]^m$, $X = [GF(2)]^r$ are state and entrance index (alphabet) respectively where $s(c)$ and $x(c)$ are defined over the set Z^K as an m and r dimensional state and entrance vectors. $\xi_\nu s(c)$ is shift operator defined as follows:

$$\xi_\nu s(c) = s(c + e_\nu); e_\nu = (0,...,0,\overset{\nu}{1},0,...,0)\ ,\ \nu=1,\ldots,k$$

$\{\Phi_\nu(c), \nu = 1,...k\}$, $\{\Psi_\nu, \nu = 1,...,k\}$ are characteristic booolean translation matries of size $m \times n$ and $m \times r$ respectively. $[GF(2)]$ Galois Field, $s(c^0) = s^0$ is the initial state vector.

If the system (1.1) defines a linear multi parameter finite sequential machinery then the machinery of the optimal discrete process is characterized by the pseudo Boolean functions given by:

$$J(x) = a's(c^L) \tag{1.2}$$

Here $a = (a_1, a_2,..., a_m)$ is the given boolean vector, $L = L_1 + L_2 + ... + L_k$ is the period of this process. Let`s define the set of administrator functions $\hat{X}$ from G_d into $[GF(2)]^r$ where i.e. $\hat{X} = \{x(c), c \in G_d\}$. As it can be seen for every $x(c) \in \hat{X}$ the number of equations in system (1.1) is more than the number of unknowns. Therefore the problem can be stated as an optimal control problem as follows .

Definition: If for $x(c) \in \hat{X}$ the system (1) has a unique solution, then we say $x(c)$ is a possible control mechanism [5].

Now for dual non-linear multi parameter finite sequential machinery we can analize the following terminal state problem.

In order for a given dual non-linear multi parameter finite sequential machinery to run from s^0 to $s^*(c^L)$ in L steps it has to exist $x(c) \in \hat{X}$ control mechanism so that, the functional in (1.2) has to have a minimal value:

$$\xi_\nu s(c) = \Phi_\nu(c)s(c) \oplus \Psi_\nu(c)x(c),\ c \in G_d \qquad \text{v= 1,...,k,} [GF(2)] \tag{1.3}$$

$$s(c^0) = s^0,$$

$$x(c) \in \hat{X}, c \in \hat{G}_d, \tag{1.4}$$

$$J(x) = a's(c^L) \to \min \tag{1.5}$$

where $\hat{G}_d = G_d \setminus \{c^L\}$.

II. NECESSARY AND SUFFICIENT CONDITIONS FOR OPTIMALITY

Assume that the system of equations in (1.1) has a unique solution ([3],[4]) and consider the problem described in the previous section where $\varphi(c) = \Phi'_\nu(c)\varphi(\xi_\nu c), c \in G_d$, ν= 1,...,k, $[GF(2)]$ is a sequential machinary and $h_\nu(c, x(c), \varphi(\xi_\nu c)) = \varphi'(\xi_\nu c)\Psi_\nu(c)x(c)$, ν= 1,...,k, $[GF(2)]$ is a boolean functional.. Then by using this functional we form the following sum.

$$\hat{h} = \sum_{L(c^l, c^L)} \sum_{\nu=1}^{k} h_\nu(c, x(c), \varphi(\xi_\nu c))\Delta_2 c_{\nu,}\ [GF(2)], \tag{2.1}$$

where Δ_2 is the initial boolean difference.

Let $\hat{L}(c^0, c^1, ..., c^L)$ be a piecewise curve connecting c^0 to c^L. In the l'th step where $0 \le l < L,$ we assign a value v_t for the piece of curve from c^l to c^{l+1}. Then the boolean sum in (2.1) takes the following form:

$$\hat{h} = \sum_{(c^l, c^L)} h_{v_t}(c^{t-1}, x(c^{t-1}), \varphi(c^t)) = \sum_{t=l+1}^{L} h_{v_t}(c^{t-1}, x(c^{t-1}), \varphi(c^t)) \quad (2.2)$$

Let's include also the Hamilton-Pontryagin functional

$$H(\varphi(c), s(c)) = \varphi(c)s(c), c \in G_d \quad (2.3)$$

which is similar to a boolean functional.

Theorem. For given point c^l where $0 \le l < L$, the control mechanism $x^0(c^l, c^{L-1}) = \{x^0(c^l), x^0(c^{l+1}), ..., x^0(c^{L-1})\}$ and a suitable orbit $s^0(c^l, c^L) = \{s^0(c^l), s^0(c^{l+1}), ..., s^0(c^L)\}$, the necessary and sufficient condition for optimality is

$$H(\varphi^0(c^l), s^0(c^l)) = \sum_{t=l+1}^{L} h_{v_t}(c^{t-1}, \varphi^0(c^t)x^0(c^{t-1})) \quad (2.4)$$

We find $\varphi^0(c^t), \quad t = l, l+1, ..., L-1$ by using equations $\varphi(c) = \Phi'_v(c)\varphi(\xi_v c), c \in G_d$, v= 1,...,k, $[GF(2)]$ with the help of the initial condition $\varphi^0(c^L) = a$.

Necessity. From (2.2) for $l+1 \le t \le L$ we have

$$\begin{aligned} H(\varphi^0(c^t), s^0(c^t)) &= \varphi^0(c^t)s^0(c^t) = \\ &= \varphi^0(c^t)\Phi_{v_t}(c^{t-1})s^0(c^{t-1}) \oplus \varphi^0(c^t)\Psi_{v_t}(c^{t-1})x^0(c^{t-1}) = \\ &= \varphi^0(c^{t-1})s^0(c^{t-1}) \oplus \varphi^0(c^t)\Psi_{v_t}(c^{t-1})x^0(c^{t-1}) = \\ &= H(\varphi^0(c^{t-1}), s^0(c^{t-1})) \oplus h_{v_t}(c^{t-1}, x^0(c^{t-1}), \varphi^0(c^t)) \end{aligned}$$

If we consider the sum from $l+1$ to L we get

$$\sum_{t=l+1}^{L} H(\varphi^0(c^t), s^0(c^T)) = \sum_{t=l+1}^{L} H(\varphi^0(c^{t-1}), s^0(c^{t-1})) \oplus \sum_{t=l+1}^{L} h_{v_t}(c^{t-1}, x^0(c^{t-1}), \varphi^0(c^t))$$

and

$$H(\varphi^0(c^L)S^0(c^L)) = H(\varphi^0(c^l), s^0(c^l)) \oplus \sum_{t=l+1}^{L} h_{v_t}(c^{t-1}, x^0(c^{t-1}), \varphi^0(c^t)) \quad (2.5)$$

Conversely we have $\Phi(s^0(c^L)) = a's^0(c^L) = \varphi^0(c^L)s^0(c^L) = H(\varphi^0(c^L), s^0(c^L))$ where $s^0(c^L)$ is an optimal state and we get $H(\varphi^0(c^L), s^0(c^L)) = 0$. This shows that equation (2.4) is correct . This finishes the proof of the necessity condition.

We handle the sufficient condition in a similar way as of necessity and from (2.5) we obtain the desired result.

Assume that the system in (1.1) is a equalibrium state. For this case we can prove the following necessary and sufficient conditions.

Theorem. In problem (1.3)-(1.5) for control mechanism $x^0(c)$ and suitable orbit $s^0(c)$ the necessary and sufficient condition for optimality is

$$(\sum_{(c^0, c^L)} \sum_{v=1}^{k} h_v(c, x^0(c), \varphi^0(\xi_v c))\Delta_2 c_v) \times \overline{(\sum_{(c^0, c^L)} \sum_{v=1}^{k} h_v(c, x(c), \varphi^0(\xi_v c))\Delta_2 c_v)} = 0, GF(2)$$

where the bar denotes the conjugate and the sum on the left is done over the piecewise curve connecting c^0 to c^L with alll possible control mechanism.

Necessity: From the solution of system of equations in (1.3) we have

$$S(c^L) = \sum_{(c^0,c^L)} \sum_{\nu=1}^{K} \hat{\phi}(c^L, \xi_\nu c)\psi_\nu(c)x(c)\Delta_2 c_\nu \quad ,$$

$$J = a's(c^L) = \sum_{(c^0,c^L)} \sum_{\nu=1}^{K} a'\hat{\phi}(c^L, \xi_\nu c)\psi_\nu(c)x(c)\Delta_2 c_\nu$$

Assume that $\varphi(\xi_\nu \mathrm{c}) = \mathrm{a}'\hat{\phi}(\mathrm{c}^{\mathrm{L}}, \xi_\nu \mathrm{c})$, $\mathrm{c} = \mathrm{c}^0, \mathrm{c}^1, ..., \mathrm{c}^{\mathrm{L}-1}$, where $\hat{\phi}(c^L, \xi_\nu c) = s(c^L)s^{-1}(\xi_\nu c)$.
So we have $\varphi^0(\mathrm{c}^{\mathrm{L}}) = \mathrm{a}$ and for the optimal control we have

$$\sum_{(c^0,c^L)} \sum_{\nu=1}^{k} h_\nu(c, x^0(c), \varphi^0(\xi_\nu c))\Delta_2 c_\nu =$$

$$= \min \sum_{(c^0,c^L)} \sum_{\nu=1}^{k} h_\nu(c, x(c), \varphi^0(\xi_\nu c))\Delta_2 c_\nu , \qquad (2.6)$$

We find the minimum on any curve connecting c^0 to c^L over all posiible $x(c)$. On this curve the equation is equal to the boolean inequality

$$\sum_{(c^0,c^L)} \sum_{\nu=1}^{k} h_\nu(c, x(c), \varphi^0(\xi_\nu c))\Delta_2 c_\nu \geq \sum_{(c^0,c^L)} \sum_{\nu=1}^{k} h_\nu(c, x^0(c), \varphi^0(\xi_\nu c))\Delta_2 c_\nu \qquad (2.7)$$

On the other hand the inequality is equal to

$$\overline{\left(\sum_{(c^0,c^L)} \sum_{\nu=1}^{k} h_\nu(c, x(c), \varphi^0(\xi_\nu c))\Delta_2 c_\nu \right)} \sum_{(c^0,c^L)} \sum_{\nu=1}^{k} h_\nu(c, x^0(c), \varphi^0(\xi_\nu c))\Delta_2 c_\nu = 0 \qquad (2.8)$$

Sufficiency: Assume we have equation (2.8). If we write

$$\varphi(\xi_\nu \mathrm{c}) = \mathrm{a}'\hat{\phi}(\mathrm{c}^{\mathrm{L}}, \xi_\nu \mathrm{c}) \quad , \mathrm{c} = \mathrm{c}^0, \mathrm{c}^1, ..., \mathrm{c}^{\mathrm{L}-1}$$

in equation (2.7) we get

$$\sum_{(c^0,c^L)} \sum_{\nu=1}^{k} a'\hat{\phi}(c^L, \xi_\nu c)\psi_\nu(c)x^0(c)\Delta_2 c_\nu \leq$$

$$\leq \sum_{(c^0,c^L)} \sum_{\nu=1}^{k} a'\hat{\phi}(c^L, \xi_\nu c)\psi_\nu(c)x(c)\Delta_2 c_\nu$$

Here $x(c)$ is any possible control. From the last equation we get $x^0(c)$ as the optimal control .

Literature

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