# ONE APPROACH TO SOLUTION OF THE PROBLEMS OF INTEGER PROGRAMMING WITH ONE RESTRICTION-EQUALITY

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Let us consider the following problem of linear programming:

$$\sum_{j=1}^{n} c_j x_j \to \max , \tag{1}$$

$$\sum_{j=1}^{n} a_j x_j = b , \tag{2}$$

$$0 \le x_j \le d_j , \ j = \overline{1,n} , \tag{3}$$

$$x_j - \text{integer}, \ j = \overline{1,n} . \tag{4}$$

It is assumed that $a_j , \ b, \ c_j , \ d_j \ (j = \overline{1,n})$ – are integer and positive.

The problem (1)-(4) refers to the class of intractable problems, i.e. the time required for solution of the problem increases exponentially at the growth of dimension.

Most scientific studies on problems of integer programming examine problems of linear programming with restriction of inequality. Evidently it connected with rigidity of restriction (2) with integer positive numbers than inequality:

$$\sum_{j=1}^{n} a_j x_j \le b$$

This study examines one approach to the solution of problem (1)-(4).

Inverse problem of aggregation (disaggregation), i.e. reduction of an integer equality to the equivalent system with lesser coefficients was solved for equality with nonnegative integer variables in [5]. Also has been proved the following theorem:

**T h e o r e m.** If for the specified integer parameter $p > 1$ exists such $t$, that satisfies condition

$$p > \max\left\{ \left|b_2 + pt - f_1\right|, \left|b_2 + pt - f_2\right| \right\},$$

then system of equations

$$\begin{cases} \sum_{j=1}^{n} a_{1j} x_j = b_1 - t, \\ \sum_{j=1}^{n} a_{2j} x_j = b_2 + pt \end{cases} \tag{5}$$

and equation (2) are equivalent, i.e. sets of integer nonnegative solutions of (2) and (5) coincide.

Here $b_2 = b \ (\text{mod}\, p)$, $b_1 = (b - b_2)/p$, $a_{2j} = a_j \ (\text{mod}\, p)$, $a_{1j} = (a_j - a_{2j})/p$, $j = \overline{1,n}$,

$$f_1 = \max_{x \in G_1(t)} \sum_{j=1}^{n} a_{2j} x_j, \quad f_2 = \min_{G_2(t)} \sum_{j=1}^{n} a_{2j} x_j,$$

$$G_1(t) = \left\{ X = (x_1, x_2, ..., x_n) \middle| \sum_{j=1}^{n} a_{1j} x_j \le b_1 - t, 0 \le x_j \le d_j, j = \overline{1,n} \right\},$$

$$G_2(t) = \left\{ X = (x_1, x_2, ..., x_n) \middle| \sum_{j=1}^{n} a_{1j} x_j \ge b_1 - t, 0 \le x_j \le d_j, j = \overline{1,n} \right\}.$$

Idea of the present approach consists of the following: Method of disaggregation lets construct equivalent system of equations (5) with lesser coefficients instead of one equation (2), then after disaggregation one can obtain equalities among coefficients which there are unities. Then accepting as a pivotal element unknown with unit coefficient one can apply simplex transformation without getting fractional coefficients. It's quite clear that value of efficiency function increases from iteration to iteration.

Let the problem (1)-(4) be the following:

$$\begin{cases} f - c_1 x_1 - c_2 x_2 - ... - c_k x_k - ... - c_n x_n = 0 \\ a_{11} x_1 + a_{12} x_2 + ... + 1 \cdot x_k + ... + a_{1n} x_n + x_{n+1} = b_1^{(1)} \\ a_{21} x_1 + a_{22} x_2 + ... + a_{2k} x_k + ... + a_{2n} x_n + x_{n+2} = b_2^{(2)} \end{cases} \quad (6)$$

Here, without upsetting commonness is supposed that variable bringing in the basis is $x_k$ and after disaggregation $a_{1k} = 1$. Besides $b_1^{(1)} = b_1 - t$, $b_2^{(1)} = b_2 + pt$

Since $x_k$ is bringing in basis, so $\min_j (-c_j) = -c_k < 0$. Then applying simplex-transformation one can exclude unknown $x_k$ from first and third equation of the system (6). Reiterating application of the simplex transformations value of $f$ can be increased, provided if among the coefficients of first equation of (6) there are negative numbers. Otherwise process of computation must be stopped.

Note that if the conditions are fulfilled, then present approach gives an optimal solution of the problem (1)-(4).

**R e m a r k.** After disaggregation of optimization problem (1)-(4) some coordinates of the optimal solutions are defined immediately. Taking that into account is obtained equivalent problem with lesser number of unknowns.

For clarity of the present approach let us consider the following example:

**E x am p l e.**

$$f = 21x_1 + 11x_2 + 10x_3 + 7x_4 + 13x_5 + 6x_6 + 11x_7 \to \max, \quad (7)$$

$$7x_1 + 4x_2 + 4x_3 + 3x_4 + 6x_5 + 4x_6 + 6x_7 = 10, \quad (8)$$

$$0 \le x_j \le 1, \ j = \overline{1,7}, \quad (9)$$

$$x_j - \text{ integers}, \ j = \overline{1,7}. \quad (10)$$

Note that the solution of the corresponding problem of relaxation is:

$x^* = (1, 0.75, 0, 0, 0, 0, 0)$, at that $f^* = 29.5$.

The system of equations equivalent to the equation (8) is the following:

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 + 2x_5 + x_6 + 2x_7 = 3 \\ x_1 + x_2 + x_3 + x_6 = 1 \end{cases} \tag{11}$$

Let us solve the problem assigned by the conditions (7), (11) and (9) by means of simplex-transformations:

I t e r a t i o n 1.

$$f_0 - 21x_1 - 11x_2 - 10x_3 - 7x_4 - 13x_5 - 6x_6 - 11x_7 = 0$$

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 + 2x_5 + x_6 + 2x_7 + x_8 = 3 \\ x_1 + x_2 + x_3 + x_6 + x_9 = 1 \end{cases}$$

$$x = (\,0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 3 \quad 1)$$

I t e r a t i o n 2.

$$f_0 \quad + 10x_2 - 10x_3 - 7x_4 - 13x_5 + 15x_6 - 11x_7 + 21x_9 = 21$$

$$\begin{cases} -x_2 - x_3 + x_4 + 2x_5 - x_6 + 2x_7 + x_8 - 2x_9 = 1 \\ x_1 + x_2 + x_3 + x_6 + x_9 = 1 \end{cases}$$

$$x = (\,1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0)$$

It's obvious that after accepting $x_1 = 1$ $x_2, x_3, x_5, x_6, x_7$ and $x_9$ get value 0. Then remains only $x_4 = 1$.

Thereby solution of the problem (7)-(10) is $x^* = (\,1, 0, 0, 1, 0, 0, 0\,)$ and $f^* = 28$.

## Literature

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