ON AN ALGORITHM OF SOLVING PROBLEMS ON OPTIMAL QUICK-ACTION WITH PHASE CONSTRAINT OF HEAT CONDUCTIVITY PROCESS

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It was determined [1] that depending on real properties of fluids and manifold there exists optimal distribution of the temperature of a stratum, at which displacement process takes place more efficiently. Maximal temperature in a stratum does not go beyond the temperature at which the damage to constructions of oil wells and coke generation are possible. In this connection there arises the necessity of solving problems on determining optimal regimes and periods of thermo-effect taking into account the constraints on the temperature of a stratum.

A problem on optimal quick-action with phase constraint for the equation of parabolic type describing heat conductivity process in an oil stratum is considered in the problem. Similar problems not taking into account phase constraint were solved in [2, 3]. A numerical algorithm of solving the problem based on solving a sequence of optimal control problems with phase constraint at the fixed time is proposed. To numerical solution to these problems, which are also of self-contained interest, methods of penalty functionals, of projection and of conditional gradient are applied. An analytical formula for the gradient of the functional with the use of finite difference method on an uneven grid was obtained. A discrete analogue of the problem was built. Numerical experiments on model problems were carried on.

Heat conductivity process in an oil stratum, the mathematical model of which has the form:

$$\frac{1}{x}\frac{\partial}{\partial x}\left(xk(x)\frac{\partial u}{\partial x}\right)+\frac{q(t)}{x}\frac{\partial u}{\partial x}-\alpha(t)u=c(x)\frac{\partial u}{\partial t}, \tag{1}$$

$$(x,t)\in G=\{x_c<x<x_r,\quad 0<t\le T\},$$

$$u(x,0)=\varphi(x),\quad x_c<x<x_r, \tag{2}$$

$$u_x(x_c,t)=g(t),\quad 0<t\le T, \tag{3}$$

$$\left(u(x,t)+k(x)u_x(x,t)\right)_{x=x_r}=v(t),\quad 0<t\le T, \tag{4}$$

where $k(x)$, $q(t)$, $c(x)$, $\varphi(x)$, $g(t)$, $\alpha(t)$ are given functions, and x_c, x_r are given values, is considered.

The following problem is stated: to find such regime of the work of a thermal source (in permissible limit) that the distribution of the temperature of a stratum, given reasoning from technological conditions, would be reached for minimal time with the indicated accuracy. At that the temperature of the stratum cannot exceed some maximal value.

Thus, it is required to find functions $v(t)$, $u(x,t)$ which satisfy conditions (1)-(4), and constraints

$$v_{\min}\le v(t)\le v_{\max},\qquad 0\le t\le T \tag{5}$$

$$u(x,t)\le u_{\max},\qquad (x,t)\in\overline{G} \tag{6}$$

so that at the given function $u^*(x)$ and constant $\delta>0$ the inequality

$$\int_{x_c}^{x_r}\left(u(x,T)-u^*(x)\right)^2 x\,dx\le\delta \tag{7}$$

would hold true for minimal time $T = T_{\min}$. Here $v_{\min}$, $v_{\max}$ and $u_{\max}$ are given values characterizing the ultimate capabilities of thermal sources and maximal permissible value of the temperature of the stratum.

Suppose that there exists such $T > 0$ and control $v(t)$ that condition (7) holds true at conditions (1)-(6). The following auxiliary problem is considered: to find such $v(t)$ for the fixed T so that the functional

$$J(v) = \int_{x_c}^{x_r} \left(u(x,T) - u^*(x)\right)^2 dx \qquad (8)$$

would take on its least possible value at conditions (1)-(6).

Control optimal on quick-action and minimal time $T_{\min}$ are found using the following algorithm [2]:

1. Such $T^{(1)}$ is chosen that the condition $\min J(v) \le \delta$ would hold true in optimal control problem (1)-(6), (8), where it is assumed that $T_{\max}^{(1)} = T^{(1)}, \quad T_{\min}^{(1)} = 0$.

Let $v(t, T^{(k)})$ be the solution to problem (1)-(6), (8) at $T = T^{(k)}$, at that $\min J(v) \le \delta$. For $k = 1,2,...$ $T^{(k+1)}$ is determined in the following way:

2. If $\min J(v) \le \delta$, then it is assumed that $T_{\max}^{(k)} = T^{(k)}$,else it is assumed that, $T_{\min}^{(k)} = T^{(k)}$.
3. $T^{(k+1)} = \dfrac{T_{\max}^{(k)} + T_{\min}^{(k)}}{2}$ is determined.
4. If $T_{\max}^{(k)} - T_{\min}^{(k)} < \varepsilon$, where $\varepsilon > 0$ is small enough, then the calculations are completed, and $T_{\min} = \dfrac{T_{\max}^{(k)} + T_{\min}^{(k)}}{2}$ is recognized as an approximate value of minimal time, and $v^*(t) = v(t, T_{\min})$, which is the solution to optimal control problem (1)-(6), (8) at $T = T_{\min}$, is recognized as a control optimal on quick-action.

Thus, the solution to problem (1)-(7) is reduced to the solution of the sequence of problems (1)-(6), (8) at fixed $T^{(k)}, \quad k = 1,2,...$. In order to solve these optimal control problems with phase constraint, method of penalty functionals is used.

Let $\{A_m\}$ be some positive sequence such that $\lim\limits_{m\to\infty} A_m = +\infty$. Construct the functional

$$\Phi_m(v) = J(v) + A_m P(v), \qquad (9)$$

where $P(v) = \int_0^T \int_{x_c}^{x_R} \left|\max\{u(x,t) - u_{\max}; 0\}\right|^2 dx dt$.

The item $A_m P(v)$ serves as a penalty for violating constraint (6), i.e. if for some $v(t)$ it does not take place, then $P(v) > 0$ and $\lim\limits_{m\to\infty} A_m \cdot P(v) = +\infty$, if (6) does not take place, then $P(v) = 0$ and the penalty item in (9) disappears.

At each $T^{(k)}$, $k = 1,2,...$ and $m = 1,2,...$ the problem on minimization of functional (9) at conditions (1)-(5) is solved with the use of methods of projection and conditional gradient. In this connection, using increment method the following formula for the gradient of functional (9) was obtained:

$$grad\ \Phi_m(v) = x_R k(x_R)\frac{\partial y(x_R,t)}{\partial x} + x_R y(x_R,t), \qquad (10)$$

where $y(x,t)$ is the solution to conjugate boundary problem:

$$-c(x)\frac{\partial y}{\partial t} = \frac{1}{x}\frac{\partial}{\partial x}\left(xk(x)\frac{\partial y}{\partial x}\right) - \frac{q(t)}{x}\frac{\partial y}{\partial x} - \alpha(t)y(x,t) +$$
$$+2A_m \max\big(u(x,t) - u_{\max};0\big), \quad x_c < x < x_R, \quad 0 \le t < T,$$
$$y(x,T) = -2\big(u(x,T) - u^*(x)\big)/c(x), \quad x_c \le x \le x_R,$$
$$q(t)y(x_c,t) - k(x_c)y_x(x_c,t) = 0, \quad 0 \le t \le T,$$
$$y(x_R,t) + \big(k(x)y_x(x,t) - q(t)y(x,t)\big)_{x=x_R}, \quad 0 \le t \le T.$$

Therefore, the solution to the optimal control problem at fixed $T^{(k)}$, $k = 1,2,...$ is reduced to the building of the sequence $v^n(t)$ by gradient projection method taking into account conditions (1)-(4), (10) and (11). The step of gradient method is determined from the condition of monotone decrease of the functional by bisection method.

For numerical realization of the algorithm finite difference method on an uneven grid is used. Boundary problems (1)-(4) and (11) at fixed $v(t)$ are solved by sweep method; the value of the functional is calculated by trapezium quadrature formula. The accuracy on the functional is controlled by the condition $J^{(n)} - J^{(n+1)} < \varepsilon_1$, where $\varepsilon_1 > 0$ is the given sufficiently small value.

The numerical experiments carried out on model problems showed the efficiency of the algorithm proposed. The method of choosing initial approaches, which speeds up the convergence of iteration process, is proposed as well.

References

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