ANALYTIC SYNTHESIS OF CONTROL IN DYNAMIC SYSTEMS WITH DELAY ARGUMENT AND FUZZY TS MODEL

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There is suggested a method of analytic synthesis of knowledge base for fuzzy controllers, providing maximum degree of stability, structure and parameters for a single class of fuzzy Tagaki-Sugeno type model and for dynamic object with delay argument.

Key words: fuzzy controllers, controlled closed loop, fuzzy Tagaki-Sugeno type model, knowledge base, degree of stability, degree of fading oscillation.

1. Introduction.

Some dynamic control systems are defined by Mamdani, or "**If**…, **Then"** type Tagaki-Sugeno (TS) model [3] or fuzzy differential equations [5].

If fuzzy dynamic control object has delay argument, then a synthesis and analysis of fuzzy controller's structure and parameters are complicated. Taking into account above mentioned, there is proposed an analytic method of fuzzy controller's base knowledge and parameters synthesis for object with maximum stability degree, defined by TS type fuzzy dynamic model with delay argument.

2. Statement of fuzzy controller's synthesis task

Control object with delay argument in technological processes with fuzziness and non-linearity is written as:

OBR_J: If a state of object x is $\widetilde{X}$ and a speed of state changing $\dot{x}$ is $\widetilde{\dot{X}}_j$, then an equation of object's movement is

$$\sum_{i=0}^{n} a_{n-i}^{j} x^{(i)}(t) = k_{ob}^{j} u(t-\tau), \quad x^{(i)}(t) = \frac{d^i x(t)}{dt^i}, \quad x^{(0)}(t) = x(t), \quad j = \overline{1,q} \tag{2.1}$$

Here $x(t)$ and $u(t)$ -are variables of condition state and control; $\widetilde{X}_j$ -fuzzy term-set of object's state coordinate (of output), $\widetilde{\dot{X}}_j$ -fuzzy term-set of object's state changing speed, j -a number of fuzzy linguistic rules; a_{n-i}^{j} $\left(i = \overline{1,n}\right)$, k_{ob}^{j} and τ -parameters and delay arguments of differential equation corresponding to j-rule, they are constant values, but different in general.

Stability and fading degree of responsible processes are considered as maximum in many technological processes and control systems of robot-technical devices.

Accounting above mentioned a control task of fuzzy dynamic object with delay argument is formulated as below.

There must be determined such fuzzy control influence or, in other words, fuzzy controller's structure that reforms fuzzy dynamic object with fuzzy delay argument, defined by TS type-model, from initial state $x(t_0) = x_0$ into final state x_s, knowledge base and parameters

$$\widetilde{u}(t) = \widetilde{\varphi}(x, k_{ta}) \quad \text{or} \quad u^j(t) = \varphi(x, k_{ta}^j), \quad j \in [1, q] \tag{2.2}$$

$$k_{ta}^j = \left(k_{t0}^j, k_{t1}^j \ldots k_{tr}^j\right), \ r \le n \ \ k_{ta}^j \in K, \quad K = \left[k_{ta}^{\min}, k_{ta}^{\max}\right] \tag{2.3}$$

that a quality indicator of system (2.1)-(2.3) be maximum for every j-linguistic rule

$$J_1^j = \max_{k_{ta} \in K}\left(-\operatorname{Re}\lambda\left(a^j, k_{ta}^j\right)\right), \quad j = \overline{1,q} \tag{2.4}$$

$$m_l^j \ge m^*, \quad m_l^j = \left|\operatorname{Re}\lambda_l(a^j, k_{ta}^j) / \operatorname{Im}\lambda_l(a^j, k_{ta}^j)\right|, \quad l \in [2, n] \tag{2.5}$$

Here J_1 is a system's stability degree and is coincided by the root's valid part that is nearest to the left semi-surface's imaginary axis in closed-loop ACS's characteristic equation $p(x,k_{ta})=0$

3. Synthesis of fuzzy controller's knowledge base, structure and parameters

It is impossible to decide above formulated task by known analytic methods, for example [3-6]. Taking into account this, there is choosed such a structure of fuzzy controller by constructing different types of feed-back connections for a synthesis of controller's structure and parameters that delay argument in ACS characteristic equation would be compensate, or it must be not used, corresponding to every linguistic rule.

A knowledge base of fuzzy controller, corresponding to object's (2.1) model is constructed as below.

Rj; **If** state coordinate x is $\widetilde{X}_j$ and state changing speed - $\dot{x}$ is $\widetilde{\dot{X}}_j$, **Then** controller's output is

$$u^j(t)=L^{-1}\{W^j_{taa},(s,k^j_{ta})\}\varepsilon(t),\quad j\in[1,q] \qquad (3.1)$$

Here L^{-1} is back Laplace's transformation, $W^j(s,k^j_{ta})$ is here a transfer function of controller's closed loop with controlling parameters $k^j_{ta}=(k^j_{t0},k^j_{t1},...k^j_{tr})$, corresponding to a linguistic j -rule.

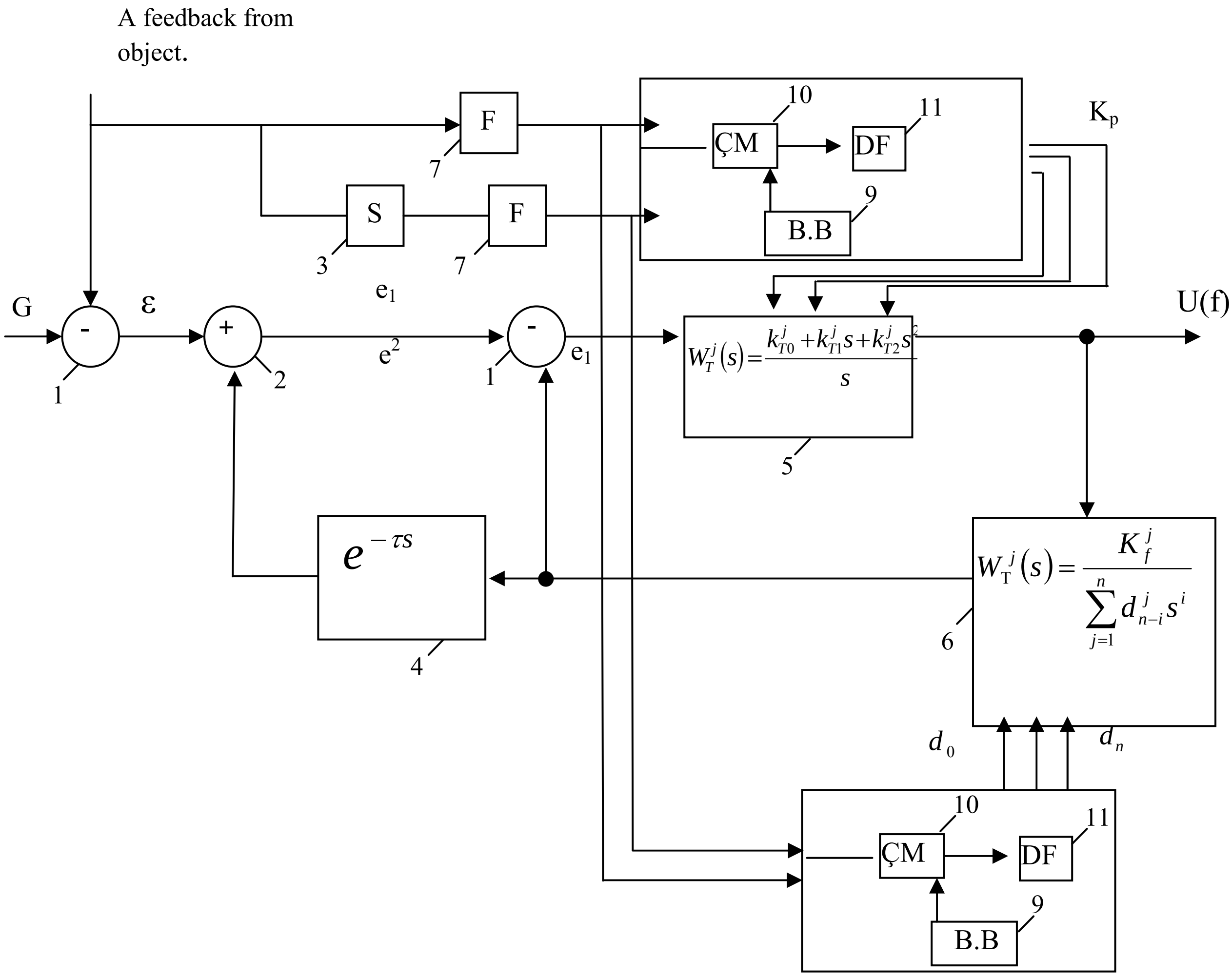

Fig.1. An architecture of controlled feedback fuzzy controller

If fuzzy (3.1) controller is corresponding to every linguistic rule, then there are constructed local positive and negative controlled feedback connections for providing compensation of delay argument in systems characteristic equation. The structure of controlled feedback fuzzy controller is proposed on fig. 1. There are 1 -comparing element, 2-summator, 3-differentiator, 4-delay block.

5-controlling blocks and is for example, standard PID, or linear controller with other non-standard rule

$$W_T^j(s) = C(s)/D(s) \tag{3.3}$$

6-is a filter block of controller feedback and transfer function

$$W_F^j(s) = k_F^j / \sum_{i=0}^{n} d_{F,n-i}^j s^i \tag{3.4}$$

8-9 are correspondingly, the fuzzy filter's knowledge bases of forward contour, fuzzy controller and local feedback, 10-fuzzy logic output, 11-defuzzification block. 10 and 11 blocks together are realizing fuzzy logic output on a base of Sugeno's algorithm [1]. 7-is a fuzzification block and formalizes fuzzy term-block, corresponding to output (x) and it's changing speed $(\dot{x})$

$$\widetilde{X}_\nu = \{x, \mu_{\nu x}(x) \mid x \in X_u\},\ \widetilde{\dot{X}}_\nu = \{\dot{x}, \mu_\nu(\dot{x}) \mid \dot{x} \in \dot{X}_u\},\ \mu_\nu \in [0,1],\ \mu_{\nu x} \in [0,1],\ \nu = 1,2,3 \tag{3.5}$$

Here X_u and $\dot{X}_u$-are, correspondingly universal sets of x and $\dot{x}$ (possible changing areas $X_u = [M^{1*}, M^{2*}]$ $\dot{X}_u = [N^{1*}, N^{2*}]$), $\mu_{\nu x}$ and μ_ν-are, correspondingly, fuzzy term-set's membership functions of object controlling value and its changing speed, and have for example a triangular form, in this case.

Parametric synthesis task of fuzzy controller, corresponding to every linguistic rule ,is decided as below. Taking into account dynamic precision and speed of (2.1) object's controllers, we choose controller structure as PID type in feed forward control loop, and, indicating every fuzzy rule by j-index, we can determine control influence) $u(t)$ after logic output and defuzzification (on a base of Sugeno algorithm) as follow.

$$U(s) = e_1(s)\sum_{j=1}^{q} \eta_j(x,\dot{x}) W_T^j(s),\quad W_T^j(s) = k_{T0}^j + k_{T1}^j s + k_{T2}^j s^2 / s\ ,\ j = \overline{1,q} \tag{3.6}$$

$$\eta_j(x,\dot{x}) = \theta_j(x,\dot{x}) / \sum_{j=1}^{q} \theta_j(x,\dot{x}),\quad \theta_j(x,\dot{x}) = \prod_{\nu,r=1}^{2} \mu_{\nu r}^j(x_r(t)), \tag{3.7}$$

Here $k_{T0}^j, k_{T1}^j, k_{T2}^j$ -are controlling parameters, that are determined as below. As right-left parts of fuzzy filter's (FF) linguistic rules in local negative and positive feedback are the same, the output of FF is determined as below

$$Y_F(s) = U(s)\sum_{i=1}^{q} \eta_i(x,\dot{x}) W_F^j(s),\quad W_F^j(s) = k_F^j / \left(\sum_{r=0}^{n} d_{n-r}^j s^r\right) \tag{3.8}$$

Positive feedback is determined on a base of the negative controlling feedback's delay in time τ_f

$$Y_{gF}(s) = \exp(-\tau_F s) Y_F(s) = \exp(-\tau_F s)\, U(s)\sum_{i=1}^{q} \eta_i(x,\dot{x}) W_F^j(s) \tag{3.9}$$

Fuzzy dynamic object's output with delay argument is determined after defuzzification as follow:

$$X(s) = \exp(-\tau s) U(s)\sum_{j=1}^{q} \eta_j(x,\dot{x}) W_{ob}^j(s) \tag{3.10}$$

Based on figure 1, we can to express ε and e_1 by following expressions:

$$e_1(s) = \varepsilon(s) + U(s)\sum_{j=1}^{q}\eta_j(x,\dot{x})W_F^j(s)[\exp(-\tau_F s) - 1] \tag{3.11}$$

Equivalent but transfer function for controller with fuzzy feedback is determined as

$$W_{TQS}(s) = Z_u(s)/(1 - Z_F(s)Z_u(s)(\exp(-\tau_F s)) + Z_F(s)Z_u(s)) \tag{3.12}$$

Transfer function of control system with TS model type fuzzy controlling feedback is determined as

$$W_{sis}(s) = \frac{Z_u(s)Z_{ob}(s)\exp(-\tau s)}{1 - Z_F(s)Z_u(s)\exp(-\tau_F s) + Z_F(s)Z_u(s) + Z_u(s)Z_{ob}(s)\exp(-\tau s)} \tag{3.13}$$

when $\tau_f = \tau$

$$W_{sis}(s) = Z_u(s)Z_{ob}(s)\exp(-\tau s)/(1 + Z_u(s)Z_{ob}(s)) \tag{3.14}$$

Thus, as it is seen from (3.14), a denominator of characteristic equation doesn't contain the transcendent part-exp $(-\tau s)$.

That is why a depending on parameters of object and controller for 2 degree object in controller's parametric synthesis task is determined, basing on proposed method in [5-6], as follows.

$$J_1^j = \left(a_1^j + k_{ob}^j k_{T2}^j\right)/3a_0^j, \quad k_{T0}^j = \frac{a_0}{k_{ob}^j}\left(J_1^j\right)^3\left(1 + \frac{1}{\left(m^j\right)^2}\right), \quad j = \overline{1,q} \quad ,$$

$$k_{T1}^j = (a_0(J_1^j)^2(3 + \frac{1}{(m^j)^2}) - a_2^j)/k_{ob}$$

We must note, that system's stability degree J_1-maximum of criterion depends on k_{T2}-differentiation coefficient of controllers except object's parameters a_1^j and k_{ob}^j, and J_1^1 is defined by boarder value $k_{T2}^{jopt} = k_{T2}^* = K_{T2}^{\max}$ for every linguistic case.

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