ON THE WEAKLY STRUCTURABLE FUZZY DYNAMIC SYSTEMS MODELLING

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In this paper the new approach to the study of weakly structurable fuzzy dynamic systems (WSDS) is presented (weakly structurable controllable dynamic system) [4-6]. Different from other approaches where the source of fuzzy uncertainty in dynamic systems is expert, this approach considers time as long as an expert to be the source of fuzzy uncertainty. This notably widens the area of studied problems. All these is connected to the incomplete, imprecise, anomal and extremal processes in nature and society, where connections between the system's objects are of subjective (expert) nature, which is caused by lack of objective information about the evolution of studied system, for example in 1) economy of developing countries, business, conflictology, sociology, medical diagnosis etc; 2) management of evacuation processes in catastrophe areas, estimation of disease spreading in epidemical regions; 3) research of complex systems of applied physics, etc. One of our purposes is to create scenarios describing possible evolution of WSDS using methods developed in this projects in the framework of expert-possibilistic theory. This includes construction of algorithms of logical-possibilistic simulations of anomal and extremal process analysis.

By the participants of the paper new mathematical apparatus[1-3] was created in 2002-2005, where main attention is paid to rapidly developing theory of fuzzy measures (some class of capacities) and integrals. Using the theory of fuzzy measures and integrals for construction of decision support systems is not a new idea. But we have chosen one part of this theory – extremal fuzzy measures [1-3], which is not much well researched. In the framework of this theory a new apparatus of extended fuzzy measures was constructed on the basis of Sujeno's upper and lower integrals. Using this apparatus new fuzzy extremal models of weakly structurable dynamic system control were created [4-6], where fuzziness is represented in time. Here the structure of time is represented by monotone extremal classes of measurable sets [1-3]. On such structures uncertainty is described by extremal fuzzy measures and problems of fuzzy-statistics of extremal fuzzy processes: identification, filtration, optimal control. Results of research are published in articles [4-6].

Short description of weakly structurably dynamic systems (WSDS):

Following the system approach of modeling of weakly structurable dynamic systems, we propose that the time structure in fuzzy dynamic system is represented in following way [3-6]:

$$\left\langle \left\{ T, F\widetilde{I}(T), \tilde{g}_T \right\}, \preceq, \oplus \right\rangle, \qquad T \equiv R_0^+, \tag{1}$$

were $\widetilde{FI}(T) \equiv \langle [0,t) \rangle_{t\geq 0}$ is σ-monotonic space of monotonically increasing measurable fuzzy time intervals; $\tilde{g}_T$ is the extended fuzzy measure on $\widetilde{FI}(T)$; $\preceq$ is the partial ordering relation in $\widetilde{FI}(T)$ and $\oplus$ is the algebraic sum operation in $\widetilde{FI}(T)$.

Suppose X $(X \neq \varnothing)$ is the set of states of some WSDS to be investigated with initial $\left(X, \tilde{\mathrm{B}}, \tilde{g}\right)$ – fuzzy measure space restriction; U $(U \neq \varnothing)$ is the set of all admissible controls acting on the system with $\left(U, \tilde{\mathrm{B}}_U, \tilde{g}_U\right)$ – fuzzy measure space restriction; Y $(Y \neq \varnothing)$ is the set of output states of the system with $\left(Y, \tilde{\mathrm{B}}_Y, \tilde{g}_Y\right)$ – fuzzy measure space restriction (Y is some transformation of X); $\tilde{P} \in \tilde{B} \otimes \tilde{F}I(T) \otimes \tilde{B}_Y$ is expert reflection process, which represents fuzzy relation describing expert fuzzy activities (estimations) of fuzzy states of the system in the output values of the system in monotonically increasing fuzzy time intervals $\{\tilde{r}_t\}_{t\geq 0}$; $\tilde{\rho} \in (\tilde{B} \otimes \tilde{FI}(T)) \otimes (\tilde{B}_U \otimes \tilde{FI}(T)) \otimes \tilde{B}$ is an fuzzy transition relation describing system state evolution in fuzzy time intervals $\{\tilde{r}_t\}_{t\geq 0}$ with control taken into account; $\tilde{u} \in \tilde{B}_U \otimes \tilde{F}I(T)$ is some binary relation describing the action of fuzzy control on the system in fuzzy time intervals $\{\tilde{r}_t\}$, $t \geq 0$ (fuzzy control process); $\tilde{Q} \in \tilde{B} \otimes \tilde{FI}(T)$ is binary fuzzy relation (fuzzy process) describing the evolution of system in time; $(s)\int$ is Sujeno extended fuzzy integral (the aggregation operator of the system objects).

Definition 1. The train

$$\left\langle \left\{ X, U, T, Y, \tilde{\rho}, \tilde{Q}, \tilde{\mathrm{P}}, \tilde{u}, (s)\int \right\} \right\rangle \tag{2}$$

is called Weakly Structurable Controllable Dynamic System (WSCDS), describing the evolution of system's states in fuzzy time intervals $\{\tilde{r}_t\}$, $t \geq 0$, using following integral equation: ($Y \equiv X$), $\forall (x,\tau) \in X \times T$:

$$\mu_{\tilde{Q}}(x,\tau) \overset{\Delta}{=} (s) \int\limits_{\mathrm{E}_{\tilde{u}}(\cdot,\tau)} \left[(s) \int\limits_{\tilde{r}_\tau} \left[(s) \int\limits_{\tilde{A}_0} \mu_{\tilde{\rho}}(x, x' u, t) \circ \tilde{g}(\cdot) \right] \circ \tilde{g}_T(\cdot) \right] \circ \tilde{g}_U(\cdot) \,, \tag{3}$$

with system's initial state $\tilde{A}_0 \equiv \mathrm{E}_{\tilde{Q}}(\cdot, 0)$.

Suppose that relation between fuzzy measure spaces $(X, \tilde{\mathrm{B}}, \tilde{g})$ and $(T, \tilde{FI}(T), \tilde{g}_T)$ can be defined using some conditional extended fuzzy measure in the following way: $\forall \tilde{r}_\tau \in \tilde{FI}(T)$:

$$\tilde{g}_T(\tilde{r}_\tau) \overset{\Delta}{=} (s)\int\limits_X \tilde{g}_t(\tilde{r}_\tau / x) \circ \tilde{g}(\cdot). \tag{4}$$

Definition 2: The process $\tilde{\mathrm{P}}$ performing in fuzzy time intervals $\{\tilde{r}_\tau\}_{\tau \geq 0}$ and defined by formula

$$\mu_{\mathrm{E}_{\tilde{\mathrm{P}}}(\cdot,\tau)}(x) = \tilde{g}_t(\tilde{r}_\tau / x) \tag{5}$$

is called the fuzzy process describing expert reflections of evolutions of WSCDS states.

Now we will consider important theorem concerning the relation between $\tilde{Q}$ and $\tilde{\mathrm{P}}$ processes.

Theorem: Suppose $\{\tilde{r}_\tau\}_{\tau\geq 0}$ and $\tilde{u}$ processes are ergodic; Let $\tilde{\mathrm{P}}$ be the fuzzy process describing expert mappings of evolutions of system states in fuzzy time intervals regarding the fuzzy measure $\tilde{g}_t(\cdot/x)$. Then:

a) The process $\tilde{Q}$ described by (3) is ergodic;

b) in the conditions of control process $\tilde{u}$ on WSCDS with initial fuzzy state $\tilde{A}_0$, the evolution of system's fuzzy states is described by fuzzy process $\tilde{Q}$ integral representation of which is the following: $\forall(x,\tau)\in X\times T$

$$\mu_{\tilde{Q}}(x,\tau)=(s)\int_{U\times T}\left\{\mu_{\mathrm{E}_u(\cdot,\tau)}(u)\wedge\mu_{\mathrm{E}_{\tilde{\rho}'}(x,\cdot,\cdot)}(u,t)\right\}\circ\tilde{g}_U\otimes\tilde{g}_{\mathrm{E}_{\tilde{\mathrm{P}}}(\cdot,\tau)(u)}(\cdot),\qquad(6)$$

where $\tilde{\rho}'$ is transition fuzzy relation

$$\mu_{\mathrm{E}_{\tilde{\rho}'}(x,\cdot,\cdot)}(u,t)\overset{\Delta}{=}(s)\int_{\tilde{A}_0}\mu_{\mathrm{E}_{\tilde{\rho}}(x,\cdot,u,t)}(x')\circ\tilde{g}(\cdot);$$

$\tilde{g}_U\otimes\tilde{g}_{\mathrm{E}_{\tilde{\mathrm{P}}}(\cdot,\tau)}$ is the composition of fuzzy measure $\tilde{g}_U$ and $\tilde{\mathrm{P}}$-extended fuzzy measure $\tilde{g}_{\mathrm{E}_{\tilde{\mathrm{P}}}(\cdot,\tau)}$ [3-5].

Using the results obtained in [3-6 we shortly described the issues of controllable extremal process modeling. The following problems have been already researched and solved:

1) We have introduced the notion of the weakly structurable controllable dynamic system (WSCDS) in case of fuzzy control action $\left(\boldsymbol{U},\tilde{\mathrm{B}}_{\boldsymbol{U}},\tilde{\boldsymbol{g}}_{\boldsymbol{U}}\right)$, where the source of uncertainty is expert reflections (expert measurements) of the system states in monotonically increasing fuzzy time intervals [3-6];

2) The notions of reflection and description processes describing WSCDS states evolution have been introduced [3-6];

3) The issues of point wise ergodicity have been studied [3];

4) The compositional representation of continuous controllable fuzzy process have been constructed correspondingly using Sujeno composition integral. Analogical model is constructed for cases of discrete time [1-3].

The following problems need further research:

1) Research of fuzzy time structure dualization;

2) Research of (1)-(6) WSDS identification problems;

3) The problem of restoring the fuzzy input-output relation of (1)-(6) WSDS;

4) The problems of (1)-(6) WSDS optimal control;

5) The problems of estimation (filtration) of (1)-(6) WSDS states;

6) The problem of estimation of pessimistic-optimistic indices of ergodicity for each problem of extremal fuzzy processes fuzzy-modeling (identification, fuzzy-optimal control, fuzzy-filtration);

7) The quantitative-basic analysis of adaptation as object of WSDS control in the environment of anomal and extremal processes;

8) construction of possibilistic-objective simulation algorithms for anomal and extremal processes based on constructed models;

9) Creation of adaptation scenarios in the environment of anomal and extremal processes using expert-possibilistic theory;

10) Development of software for universal library implementing the WSDS structure and decision support methods; Creation of decision-support systems for real applications.

Literature

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