NUMERICAL METHOD OF THE DECISION OF AN INCORRECT PROBLEM TWO PHASE FLOW IN A LAYER

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It is known, that at modelling processes of replacement of oil from a layer water traditionally uses the system of the equations including the differential equations of indissolubility of filtrational streams of oil and water, the equation of movement of liquids, the equations of a condition of the porous environment and liquids [1]. Thus the geometrical configuration of a layer, and also property of breed and liquids are considered known. The system of the equations is supplemented with the initial and boundary conditions describing an initial condition of a layer and interaction of a layer with surrounding area. Usually boundary conditions are set concerning pressure or the charge of liquids (or their combinations) on chinks and on external border of a layer.

However it is necessary to note very important circumstance concerning a boundary condition on external border of a layer. The matter is that the basic sources of the information on the processes occuring in a layer by development, operational and delivery chinks where pressure and charges of liquids are accessible to direct measurements are. However the hydrodynamical processes occuring on external border of a layer by development are not accessible to direct supervision. Therefore exact representation of a condition on external border of a layer practically is not obviously possible.

In this connection for practice of development of oil deposits very much the great value has a question of modelling of process of replacement of oil from a layer water only on the basis of the information received from chinks.

Let it is horizontal the located bidimentional oil layer with constant capacity H in circular area, $\overline{G} = \{0 \le r \le R, 0 \le \theta \le 2\pi\}$, it is developed by system of wells in a mode . Water, finished through delivery chinks in a layer, does not mix up with oil and in a layer two-phase flow is formed. We shall assume, that one of delivery wells is in the center of a layer, and other wells are located in any points of a layer. Whereas the sizes of chinks there is less than size of a layer, it is possible to neglect the sizes of chinks, representing their dot drains with the capacities equal to charges of real wells [1]. Then representing wells, except for central, the dot drains described by function Dirac, the mathematical model of two phase flow of incompressible liquids in a layer can be presented in the following kind

$$H\frac{\partial mS_w}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rH\frac{kk_w}{\mu_w}\frac{\partial P}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(H\frac{kk_w}{\mu_w}\frac{\partial P}{\partial\theta}\right) - \frac{1}{r}\sum_{l=1}^{L-1} Q_l^w \delta(r-r_l, \theta-\theta_l),$$

$$H\frac{\partial mS_o}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rH\frac{kk_o}{\mu_o}\frac{\partial P}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(H\frac{kk_o}{\mu_o}\frac{\partial P}{\partial\theta}\right) - \frac{1}{r}\sum_{l=1}^{L-1} Q_l^o \delta(r-r_l, \theta-\theta_l), \qquad (1)$$

$$S_w + S_o = 1,$$

Where $P(r,\theta,t)$ is pressure in a layer, $k(r,\theta)$ is absolute permeability of a layer, $k_0(S_w), \mu_o$ is relative phase permeability and viscosity of oil, $k_w(S_w), \mu_w$ is corresponding designations for water, S_w is a water-saturation, S_o is a petrosaturation, $m = m_0 + \beta(P - P_0)$ is porosity of a layer, β is factor of elasticity of a layer, Q_l^o is production l wells on oil, Q_l^w is production l-wells on water, L is the general number of wells, $\delta(r-r_l, \theta-\theta_l)$ is function Dirac, (r_l, θ_l) are coordinates l-wells.

Production a delivery well it is considered negative, and production an operational well it is considered positive. System (1) we shall lead to system of two equations concerning a water-saturation $S = S_w$ and pressure P

$$\frac{\partial P}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma\frac{\partial P}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\sigma\frac{\partial P}{\partial\theta}\right)-\frac{1}{r}f(r,\theta,t), \tag{2}$$

$$\frac{\partial mS}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial P}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\lambda\frac{\partial P}{\partial\theta}\right)-\frac{1}{r}\varphi(r,\theta,t), \tag{3}$$

Where

$$\sigma=\frac{k}{\beta}\left(\frac{k_o}{\mu_o}+\frac{k_w}{\mu_w}\right),\ \lambda=\frac{kk_w}{\mu_w},\ f=\sum_{l=1}^{L-1}(q_l^w+q_l^o)\delta(r-r_l,\theta-\theta_l),\ \varphi=\sum_{l=1}^{L-1}q_l^w\delta(r-r_l,\theta-\theta_l),$$

Initial distributions of pressure and water-saturation, and also the charge and saturation of water to the central chink it is counted set. Then for system (2), (3) we shall have the following conditions

$$P\big|_{t=0}=\chi(r,\theta)\ , \tag{4}$$

$$S\big|_{t=0}=\psi(r,\theta)\ , \tag{5}$$

$$2\pi r\sigma\frac{\partial P}{\partial r}\Big|_{r=r_w}=q_0^w, \tag{6}$$

$$S\big|_{r=r_w}=S_w. \tag{7}$$

It is obvious, that for a correctness of a problem (2) - (7) it is necessary to have conditions of periodicity of the decision of system (2) - (7) and a condition on external border of a layer. Conditions of periodicity of the decision can be written down as

$$P(r,0,t)=P(r,2\pi,t), \tag{8}$$

$$\frac{\partial P}{\partial\theta}\Big|_{\theta=0}=\frac{\partial P}{\partial\theta}\Big|_{\theta=2\pi}, \tag{9}$$

$$S(r,0,t)=S(r,2\pi,t). \tag{10}$$

However in connection with that pressure and the charge of liquids upon external border of a layer are not accessible to direct measurements and cannot be adjusted, formulate the boundary condition corresponding to interaction of a layer with surrounding area it is not obviously possible.

Let's assume, that on a well which is taking place in the center of a layer, alongside with the charge of water pressure on face is simultaneously set also. Then instead of a condition on external border of a layer we shall have an additional condition at $r=r_w$

$$P\big|_{r=r_w}=p_w(t). \tag{11}$$

Thus, the problem about a presence of distribution of pressure $P(r,\theta,t)$ and a water-saturation $S(r,\theta,t)$ during development of a layer is reduced to the decision of system (2), (3) at performance of conditions (4) - (11).

The problem (2) - (11) concerns to a class of boundary invers problems [2,3]. For the numerical decision of a problem (2) - (11) we shall enter uniform finite scheme a grid on a variable t on a piece $0\le t\le T$

$$t_n=n\Delta t\quad ,n=0,1,2.......N_t$$

With step $\Delta t=T/N_t$ and according to a method komponent splittings of [4] problems (2) - (11) in each piece $[t_n,t_{n+1}]$ it is reduced to system of consistently decided one-dimensional problems

$$\frac{1}{2}\frac{\partial P}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma\frac{\partial P}{\partial r}\right)-\frac{1}{2r}f,\qquad t_n<t\le t_{n+\frac{1}{2}} \tag{12}$$

$$P\big|_{r=r_w}=p_w(t), \tag{13}$$

$$2\pi r\sigma\frac{\partial P}{\partial r}\Big|_{r=r_w}=q_w^0, \tag{14}$$

$$\frac{1}{2}\frac{\partial P}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\sigma \frac{\partial P}{\partial \theta}\right) - \frac{1}{2r} f \,, \qquad t_{n+\frac{1}{2}} < t \le t_{n+1} \tag{15}$$

$$P\big|_{\theta=0} = P\big|_{\theta=2\pi} \,, \tag{16}$$

$$\frac{\partial P}{\partial \theta}\Big|_{\theta=0} = \frac{\partial P}{\partial \theta}\Big|_{\theta=2\pi} \,. \tag{17}$$

$$\frac{1}{2}\frac{\partial mS}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial P}{\partial r}\right) - \frac{1}{2r}\varphi \,, \qquad t_n < t \le t_{n+\frac{1}{2}} \tag{18}$$

$$S\big|_{r=r_w} = S_w \,, \tag{19}$$

$$\frac{1}{2}\frac{\partial mS}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\lambda \frac{\partial P}{\partial \theta}\right) - \frac{1}{2r}\varphi \,, \qquad t_{n+\frac{1}{2}} < t \le t_{n+1} \tag{20}$$

$$S\big|_{\theta=0} = S\big|_{\theta=2\pi} \tag{21}$$

$$n = 0,1,2,\dots N_t - 1 \,,$$

$$P(r,\theta,0) = \chi(r,\theta) \,, \tag{22}$$

$$S(r,\theta,0) = \psi(r,\theta) \,. \tag{23}$$

From the circuit of splitting follows, that for calculation $P(r,\theta,t_{n+1})$ and $S(r,\theta,t_{n+1})$ on set, $P(r,\theta,t_n)$, $S(r,\theta,t_n)$ $n = 0,1,2,\dots N_t - 1$ it is necessary:

1. To find $P(r,\theta,t_{n+1/2})$ - the decision of an one-dimensional invers problem (12) - (14) at $t = t_{n+1/2}$.
2. On calculated $P(r,\theta,t_{n+1/2})$ to find $P(r,\theta,t_{n+1})$ - the decision of a problem (15) - (17) at $t = t_{n+1}$.
3. On calculated $P(r,\theta,t_{n+1})$ to find $S(r,\theta,t_{n+1/2})$ - the decision of a problem (18) - (19) at $t = t_{n+1/2}$.
4. To find $S(r,\theta,t_{n+1})$ - the decision of an one-dimensional problem (20) - (21) at $t = t_{n+1}$.

For the numerical decision of system of one-dimensional problems (12) - (23) we use a method of final differences. With this purpose we shall enter a non-uniform grid into areas $\overline{G}$

$$\overline{\omega} = \left\{(r_i,\theta_j) \in \overline{G},\ r_i = r_{i-1} + \Delta r_i,\ 1 \le i \le N_r,\ r_0 = r_w,\ r_{N_r} = R,\ \theta_j = \theta_{j-1} + \Delta\theta_j, 1 \le j \le N_\theta,\ \theta_0 = 0,\ \theta_{N_\theta} = 2\pi\ \right\}$$

and a problem (12) - (14) it is approximated on a grid $\overline{\omega}$

$$r_i \frac{p_{i,j}^{n+1/2} - p_{i,j}^n}{\Delta t} = \frac{1}{\Delta r_{i+1/2}}\left(r_{i+1/2}\sigma_{i+1/2,j}\frac{p_{i+1,j}^{n+1/2} - p_{i,j}^{n+1/2}}{\Delta r_{i+1}} - r_{i-1/2}\sigma_{i-1/2,j}\frac{p_{i,j}^{n+1/2} - p_{i-1,j}^{n+1/2}}{\Delta r_i}\right) - \frac{f_{i,j}^n}{2}, \qquad 1 \le i \le N_r - 1$$

$$p_{0,j}^{n+1/2} = p_w^{n+1/2} \,, \quad r_w \frac{p_{0,j}^{n+1/2} - p_{0,j}^n}{\Delta t} = \frac{2}{\Delta r_1}\left(r_{1/2}\sigma_{1/2,j}\frac{p_{1,j}^{n+1/2} - p_{0,j}^{n+1/2}}{\Delta r_1} - \frac{q_0^w}{2\pi}\right) - \frac{f_{0,j}^n}{2}$$

The received system of the equations we shall write down as

$$a_{i,j}\,p_{i-1,j}^{n+1/2} - c_{i,j}\,p_{i,j}^{n+1/2} + b_{i,j}\,p_{i+1,j}^{n+1/2} = -d_{i,j} \,, \qquad 1 \le i \le N_r - 1 \,, \tag{24}$$

$$p_{0,j}^{n+1/2} = \chi_j\,p_{1,j}^{n+1/2} + \nu_j \,, \tag{25}$$

$$p_{0,j}^{n+1/2} = p_w^{n+1/2} \,. \tag{26}$$

Let's notice, that the system of the linear algebraic equations (24) - (26) has a three-diagonal matrix, hence, if will be somehow determined $P_{N_r,j}^{n+1/2}$, the decision of the given system can be found a steady method of prorace [4]. Therefore a condition (26) it is used for definition of boundary value $P_{N_r,j}^{n+1/2}$. The decision of system (24), (25) we shall present as

$$p_{i,j}^{n+1/2} = \xi_{i+1,j}\, p_{i+1,j}^{n+1/2} + \eta_{i+1,j}\,, i = 0,1,2,\dots,N_r - 1, \qquad (27)$$

Where $\xi_{i+1,j}, \eta_{i+1,j}$ - unknown while factors. Writing down similar expression for $P_{i-1,j}^{n+1/2}$, we shall find

$$p_{i-1,j}^{n+1/2} = \xi_{i,j}\xi_{i+1,j}\, p_{i+1,j}^{n+1/2} + \xi_{i,j}\eta_{i+1,j} + \eta_{i,j}\,.$$

Substituting the received expressions for $P_{i,j}^{n+1/2}, P_{i-1,j}^{n+1/2}$ in the equation (24), we shall receive the following formulas for definition of factors $\xi_{i+1,j}, \eta_{i+1,j}$

$$\xi_{i+1,j} = b_{i,j} / (c_{i,j} - \xi_{i,j} a_{i,j}), \qquad \eta_{i+1,j} = (a_{i,j}\eta_{i,j} + d_{i,j}) / (c_{i,j} - \xi_{i,j} a_{i,j}), \quad i = 1, 2, \dots, N_r - 1.$$

And initial values of these factors we find from the requirement of equivalence of a condition (25) at i=0 the equation (27) $\xi_{1,j} = \chi_j, \quad \eta_{1,j} = \nu_j$.

After factors $\xi_{i,j}, \eta_{i,j}$ are found for all $i = \overline{1, N_r}$ it is possible to define boundary value $P_{N_r,j}^{n+1/2}$. For this purpose it is enough to write down dependence between $P_{0,j}^{n+1/2}$ and $P_{N_r,j}^{n+1/2}$ in an obvious kind. We shall write down the equation (27) at *i*=0: $p_{0,j}^{n+1/2} = \xi_{1,j}\, p_{1,j}^{n+1/2} + \eta_{1,j}$.

Now having substituted here expression $p_{1,j}^{n+1/2} = \xi_{2,j}\, p_{2,j}^{n+1/2} + \eta_{2,j}$, we shall have

$$p_{0,j}^{n+1/2} = \xi_{1,j}(\xi_{2,j}\, p_{2,j}^{n+1/2} + \eta_{2,j}) + \eta_{1,j}\,.$$

Further, substituting in last equation of expression for $p_{2,j}^{n+1/2}, p_{3,j}^{n+1/2}, \dots, p_{N_r-1,j}^{n+1/2}$, , we shall receive the formula in which $P_{0,j}^{n+1/2}$ it is expressed through $P_{N_r,j}^{n+1/2}$,

$$p_{0,j}^{n+1/2} = p_{N_r,j}^{n+1/2} \prod_{i=1}^{N_r} \xi_{i,j} + \sum_{i=2}^{N_r} \eta_{i,j} \prod_{m=1}^{i-1} \xi_{m,j} + \eta_{1,j}\,.$$

From here it is possible to find boundary value $P_{N_r,j}^{n+1/2}$,

$$p_{N_r,j}^{n+1/2} = \frac{p_{0_r,j}^{n+1/2} - \sum_{i=2}^{N_r} \eta_{i,j} \prod_{m=1}^{i-1} \xi_{m,j} - \eta_{1,j}}{\prod_{i=1}^{N_r} \xi_{i,j}}\,.$$

Having defined $P_{N_r,j}^{n+1/2}$, the decision of system (24) - (26) can be found under the recurrent formula (27), since $i = N_r - 1$. Thus, having defined the decision of an one-dimensional inverse problem (12) - (14) at $t = t_{n+1/2}$ it is possible to proceed to the decision of problems (15) - (23). We shall note, that the decision of a problem (15) - (17) certainly finite scheme is resulted by a method in [5]. And for the numerical decision of problems (18), (19) and (20), (21) it is possible to use obvious certainly – finite scheme with definition phase perlabilities « upwards on a stream ».

Literature

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