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ON PROBLEMS OF STATISTICAL ACCEPTANCE CONTROL

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Mathematical methods of statistical acceptance control (SAC) of ready products are one of the important fields of application of mathematical statistics in industry. SAC of a group (totality) of ready products is carried out, on the whole, by a qualitative or quantitative indication.

If SAC is carried out by a qualitative indication, a product is classified as a suitable (good) or defect (bad) one. In the case when requirements to quality of the checking totality are too big, existence of if only one defect product is inadmissible. In these cases detection of if only one product under sampling control means that the solution about rejection of all products totality must be accepted. In the general case it is fixing the sample volume n, i.e. the number of products selected for checking, and it is given the acceptance number c, i.e. such a number c that the totality is accepted as a suitable if the number of defect products in the sample d is no more than c, and if $d > c$, the totality is rejected. Inadmissibility of defect products means that $c = 0$. Plans of sampling control with the fixed sample volume n and the acceptance number c are denoted shortly (n, c) ($c = 0$ for plans of non-defect control). Plans of the type (n, c) are studied in details in works by Kolmogorov, Belyaev, Sirajdinov [1]-[3].

Plans of sampling control of a represented totality of products by a quantitative indication are more general in comparison with SAC by a qualitative indication. In this case it is considered that each product is characterized by some quantitative value ξ (for example, ξ can mean the bearing diameter, thread tensile strength, service life of an electric bulb, quantity of spots on the fruit's surface and so on). From the formal point of view, ξ is considered as a random variable with continuous or discrete distribution. Plans for SAC by a quantitative indication are more general since for the given limit value T, product can be considered as a suitable if $\xi \le T$ and a defect if $\xi > T$.

Let a quantitative characteristic of products ξ is a random variable with continuous distribution,

$$F(x) = \int_{-\infty}^{x} p(u)du, \quad \mu = E\xi, \quad \sigma^2 = D\xi .$$

Consider the case of SAC $(\bar{x}, \sigma^2)$ in which the average μ is unknown and the variance σ^2 is known. A sample of volume n is selected from the totality of volume N, and $\xi_1, \xi_2, ..., \xi_n$ are results of checking of these n products, respectively,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} \xi_i .$$

If $\bar{x} \le k$, the totality is accepted as a suitable, and if $\bar{x} > k$, it is rejected. In the rejected totality all products are subjected to overall control. To count common lose related to realization of SAC, introduce the following cost characteristics (parameters): let a be a damage of a defect product in accepted party, b be a damage of a defect product in rejected party, c be a cost of checking of a product in the sample, l be a cost of checking of a product under total checking.

A sample of volume n is taking from the totality of products of volume N. If in result of checking the totality is accepted, then the waiting lose

$$L_a(p,n) = Npa + nc$$

In the case of rejection (the rejected totality is subjected to overall control) the waiting lose

$$L_r(p,n) = Npb + (N-n)l + nc\,.$$

Here p is the part of defect products in the totality,

$$p = \boldsymbol{P}(\xi > T) = 1 - F(t)\,.$$

Let

$$Na = A, \quad Nb = B, \quad (N-n)l = L\,.$$

Then

$$\begin{aligned} L_a(p,n) &= Ap + nc, \\ L_r(p,n) &= Bp + L + nc. \end{aligned} \tag{1}$$

If $a > b + c$ (in this case there is a point in SAC), then for

$$p = p_0 = \frac{L}{A-B} = \frac{l}{a-b} \tag{2}$$

the equality

$$L_a(p_0,n) = L_r(p_0,n) \tag{3}$$

is valid.

Naturally, the part $p = p_0$ is called "*the indifference part*".

Let $W(p,n,k)$ be an operative characteristic of SAC, i.e. $W(\cdot,\cdot,\cdot)$ is the probability of the totality acceptance. Then for $p = p_0$ and any n and k one can accept or reject the checking totality with the same probability, i.e.

$$W(p_0,n,k) = \frac{1}{2}\,, \tag{4}$$

and we have the possibility to define the value of k from the equality (4).

The mean lose connected with SAC

$$L(p,n,k) = ApW(p,n,k) + (Bp+L)\left[1 - W(p,n,k)\right] + nc\,.$$

Represent the function $L(\cdot,\cdot,\cdot)$ in the following form

$$L(p,n,k) = L_m(p) + R(p,n,k)$$

where

$$L_m(p) = \min\{Ap, Bp+L\}\,,$$

$$R(p,n,k) = \begin{cases} (A-B)(p_0-p)\left[1-W(p,n,k)\right] + nc & \text{if } \; p \le p_0, \\ (A-B)(p-p_0)W(p,n,k) + nc & \text{if } \; p \le p_0. \end{cases} \tag{5}$$

It is clear that $L_m(p)$ is represented "the inevitable lose", and therefore it is naturally to call $R(\cdot,\cdot,\cdot)$ "the remainder lose". In the case when the part of defect products can be considered as a random variable with the known distribution on [0,1], problems of organization of SAC were solved by A. Hald [4]. But in the general case when there is no information about the variable $p \in [0,1]$, SAC is reduced to the following: to find optimal $n = n_0$ and $k = k_0$ such that

$$\min_{k,n} \max_{p} R\left(p,n,k\right) = \max_{p} R\left(p,n_0,k_0\right) .$$

In [6], it is found the asymptotical expression for the volume of the optimal SAC $n_0\left(N\right)$ ($N \to \infty$) under the condition that the number of defect products in the totality has the binomial distribution. In [7], this problem is solved in the case when a qualitative indication of products ξ has the normal distribution.

In the present work, the problem on choice of the optimal plan of SAC is solved for the case when ξ has a continuous distribution with the density function of the form

$$p\left(x\right) = \frac{1}{\sigma} f\left(\left|\frac{x-\mu}{\sigma}\right|\right), \quad \mu \in \mathbf{R}, \quad \sigma^2 > 0$$

where $f\left(x\right)$ does not depend on μ and σ (i.e. μ is the translation parameter, σ is the scale parameter).

For a plan of the type $\left(\overline{x},\sigma^2\right)$, i.e. for the case when μ is unknown and σ^2 is known the following theorem is proved.

Theorem. *For the optimal sample volume* $n_0 = n_0\left(N\right)$ *and for SAC* $\left(\overline{x},\sigma^2\right)$*, the relation*

$$\lim_{N\to\infty} \frac{n_0\left(N\right)}{N^{2/3}} = 0.193\left[\frac{a-b}{c} f\left(v_0\right)\right]^{2/3}$$

holds. Here v_0 *is the solution of the equation*

$$\int_0^v p\left(u\right)du = 1 - p_0 .$$

This theorem generalizes results of [7] essentially.

Literature

1. A.N. Kolmogorov. Unbiased estimates. (In Russian) Izvestiya AN SSSR. Ser. Matematika. 1950, Vol.14, No.4, 303-326
2. Yu.K. Belyaev. Probabilistic Methods of Sampling Control. (In Russian) Nauka, Moscow (1975) 397 p.
3. S.Kh. Sirajdinov. Proceedings of Institute of Mathematics and Mechanics of Academy of Sciences of Republic of Uzbekistan. (In Russian) Vol.15, Tashkent (1955), 11-34.
4. A. Hald. Mathematical Statistics with Technical Applications. (in Russian) IL, Moscow (1956) 583 p.
5. W. Feller. An Introduction to Probability Theory and its Applications. Vol.2. Second Edition. New York – London – Sydney – Toronto (1971) 597 p.
6. Van der Warden. Sampling Inspection as a minimum lose problem. Annals of Math. Stat. Vol. 31, No. 2 (1960), 1-13.
7. K. Stange. Die Berechnung Wirtscaftlicher Plane fьr messende Prьfung. Metrica, Vol.8, Fose 1 (1964), 101-117.