EVENTOLOGICAL FORMALIZATION OF LINGUISTIC VARIABLE

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Ability of the human to estimate the information is most brightly shown in using of natural languages. Using words of a natural language for valuation qualitative attributes, for example, the person pawns uncertainty in form of fuzziness in itself estimations. Fuzzy sets, fuzzy judgments, fuzzy conclusions takes place there and then, where and when the reasonable subject exists and also is interested in something. The fuzzy sets theory has arisen as the answer to an illegibility of language the reasonable subject speaks. Language of a reasonable subject is generated by fuzzy events which are created by the reason and which are operated by the mind. The theory of fuzzy sets represents an attempt to find such approximation of fuzzy grouping which would be more convenient, than the classical theory of sets in situations where the natural language plays a significant role. Such theory has been offered by known american mathematician L. Zadeh in 1965 [1].

One of the fundamental concepts entered by L. Zadeh, is the concept of a linguistic variable [2, 3]. It is the five objects,

$$\langle L,\ T(L),\ U,\ G,\ S\rangle, \tag{1}$$

where L is a name of a variable; $T(L)$ is a set of its values (a term-set), being syntagmas[1]; U is an universal set; G is a syntactic rule (it is a context-free grammar more often), using which we can form syntagmas $A,\ B,\ldots \in T(L)$; M is a semantic rule, using which to each syntagma $A \in T(L)$ is attributed its value, being by fuzzy set in universal set U. Syntactic procedure G allows not only to operate with elements of the term-set $T(L)$, but also to generate new syntagmas (terms) by means of links *"and", "or"*, negation *"not"*, linguistic gradation, such as *"very much"*, *" more or less "*, *"essentially"*, etc. A semantic rule S defines a way of calculation of sense of any term from the set $T(L)$. Zadeh has suggested to formalize atomic terms by fuzzy sets $\tilde{A}$ in U. Links, uncertainty and negation are treated as operators which alter sense of primary terms in the special way independent of a context [2]. Now in the theory of fuzzy sets for formalization of links *"and"*, *"or"* t-norms and t-conorm are used widely. These operations are well enough studied and underlie of many formal constructions of fuzzy logic.

Zadeh's theory is the most successfully applied there and then, where and when the fuzziness is generated by presence of the person and of its reason. This problem is the main for one of directions of the eventology (the theory of the random fuzzy events), which has arisen within the limits of the probability theory and which pursues the unique purpose to describe eventologically a movement of the reason. Enentology is a theory which unexpected for a foreign sight applies for creation of original and rather natural mathematical language for discussion of the general theoretical bases of uncertainty. The eventological substantiation of the theory of fuzzy sets of Zadeh was offered by O.J.Vorob'ov in 2004. General principles of the theory of fuzzy events using the eventological language are stated in [4].

Here the eventological formalization of a linguistic variable is offered.

The five objects,

$$\langle L,\ X,\ \Omega,\ G_E,\ S_E\rangle, \tag{2}$$

is called an eventological linguistic variable (E-linguistic variable). L is a name of variable; X is a set of names of events; Ω is a set of elementary events; G_E is a syntactic rule (context-free grammar); S_E is an eventological semantic rule.

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[1] Any part of the offer making sense and formed according to grammatical rules is a syntagma.

It is necessary to notice, that the set of names of events X is equivalent to the set of atomic syntagmas *T(L)* from (1), the set of elementary events and Ω coincides with the universal set U, the context-free grammar G and the G_E are equivalent. The essential distinction between (1) and (2) consists in the definition of the semantic rule S_E based on concept of fuzzy experiment

Two phenomena, namely uncertainty and a fuzziness which scientific importance has increased mainly in the last century are discussed in [3] in detail enough. Both these concepts characterize situations in which we consider the phenomena surrounding us. They concern to volume of the knowledge having (or able to be) in our disposal which, however are limited. The phenomenon of uncertainty arises because of lack of the knowledge concerning occurrence of some event. It meets till the moment of carrying out of some experiment which result is unknown for us. The mathematical model of the phenomenon of uncertainty is based on the device of probability theory. The fuzziness concerns to a way of the description of the event and does not consider a question about it appearance. The mathematical model of the phenomenon of the fuzziness is based on the device of the theory of fuzzy sets. In [3] it is noted, that these phenomena are represented like two sides supplementing each other of the most general phenomenon which authors name *undeterminancy*. From our point of view undeterminancy, considered in [3] is very well described by the fuzzy experiment offered in work by O.Yu.Vorob'ov [4].

A fuzzy experiment is inconceivable without participation of set of the individual reasonable subjects making an integral part of fuzzy experiment. Each individual reasonable subject is the participant of fuzzy experiment and has his own opinion on, whether it is possible to characterize event in fuzzy experiment by the given syntagma or not. All of their opinions form *fuzzy event* as an outcome of fuzzy experiment.

Let's illustrate formalization of a E-linguistic variable "*Age"*. Let's consider fuzzy experiment in which participate M reasonable subjects. Each reasonable subject $\mu \in M$ is characterized by syntagmas (names) from set X. $X=\{x, y\}$, where x is "*the young man*", y is *"the young woman"* for example. Then the intercept [0, 80] of the real axis acts as a set of elementary events Ω. The age of the concrete person corresponds to an elementary event $\omega \in \Omega$. Let (Ω, F, $\boldsymbol{P}$) is probabilistic space. Let's define a *matrix of the selected random events* [4] as a set of events $X_M=\{x_\mu, x \in X, \mu \in M\}$, where $x_\mu \in F$ are measurable random events concerning algebra. Thus, each pair $(x,\mu) \in X \times M$ defines one random event $x_\mu \subseteq \Omega$. In our example random event $x_\mu = [a_\mu, b_\mu]$ is a judgment of reason μ, in which it correlates an age intercept $[a_\mu, b_\mu] \subseteq \Omega$ to a syntagma (name) (tab. 1).

Tab.1. Matrix of the selected events

x	$\cdots$	$[a_\lambda, b_\lambda]$	$\cdots$	$[a_\mu, b_\mu]$	$\cdots$
y	$\cdots$	$[c_\lambda, d_\lambda]$	$\cdots$	$[c_\mu, d_\mu]$	$\cdots$
	$\cdots$	λ	$\cdots$	μ	$\cdots$

The eventology [4] defines M-fuzzy event as set of usual Kolmogorov's events, when $\mu \in M$. Thus, sets of elements of rows of a matrix of selected events $x_M = \{x_\mu, \mu \in M\}$ define fuzzy events $\widetilde{x} = x_M$ and form the set of M-fuzzy events $\widetilde{X} = \{\widetilde{x}, x \in X\}$. In our example we consider a set of M-fuzzy events $\widetilde{X} = \{\widetilde{x}, \widetilde{y}\}$, generated by the set X, where $\widetilde{x} = \{[a_\mu, b_\mu], \mu \in M\}$ is *"the young man"* and $\widetilde{y} = \{[c_\mu, d_\mu], \mu \in M\}$ $\mu \in M$ *"the young woman"*.

In the Zadeh's theory the fuzzy set is defined by membership function on U, which kind cannot be deduced theoretically from more simple concepts, and it is established in each problem, proceeding from external in relation to the theory of consideration. Eventology unequivocally defines *eventological membership function* of fuzzy event as the indicator of fuzzy event

$$\mathbf{1}_{\tilde{x}}(\omega)=\frac{1}{|M|}\sum_{\mu\in M}\mathbf{1}_{x_\mu}(\omega),\quad x\in X, \tag{3}$$

where $\mathbf{1}_{x_\mu}(\omega)$ - indicators of "usual" events x_μ which make sets of events $\tilde{x}$. In our example (3) for corresponding fuzzy events from the X it will be transformed to a kind (fig. 1):

$$\mathbf{1}_{\tilde{x}}(\omega)=\frac{1}{|M|}\sum_{\mu\in M}\mathbf{1}_{[a_\mu,b_\mu]}(\omega),\qquad \mathbf{1}_{\tilde{y}}(\omega)=\frac{1}{|M|}\sum_{\mu\in M}\mathbf{1}_{[c_\mu,d_\mu]}(\omega).$$

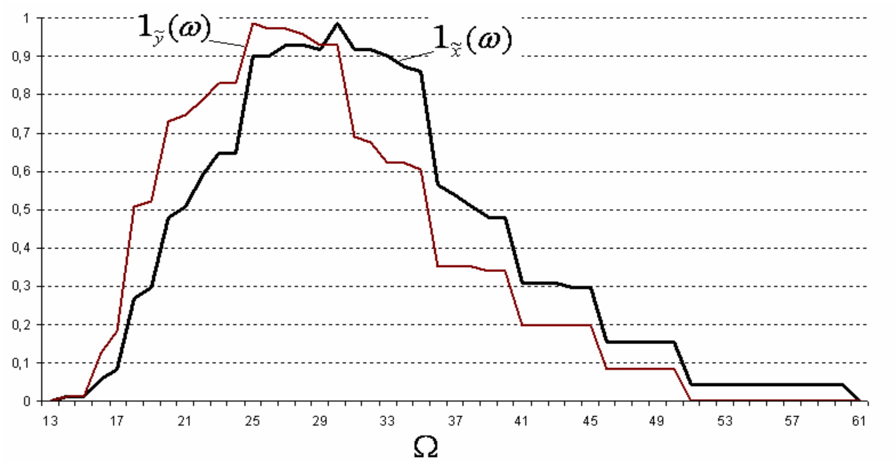

Fig.1. Eventological membership functions of fuzzy events $\tilde{x}$ - *"the young man"* and $\tilde{y}$ - "*the young woman"*, constructed as a result of fuzzy experiment in which participated $|M| = 71$ reasonable subjects.

It is necessary to notice, that membership functions of fuzzy sets, constructed by a method of expert estimations, under certain conditions coincide with the indicator of corresponding fuzzy event.

Thus, we formalized atomic syntagmas *"the young man"* and *"the young woman"* by E-linguistic variable *L: = "Age"* by fuzzy events $\tilde{x}$ and $\tilde{y}$. Links, uncertainty and the negation defined by the syntactic procedure G_E, are formalized by means of set-operations above set of M-fuzzy events $\tilde{X}=\{\tilde{x},x\in X\}$, generated by the set X [4] according to Minkovsky. The basic eventological theorem about fuzzy events [4] offers the formula for calculation of the indicator (eventological membership functions) of any set-operation above set of fuzzy. Let's consider a syntagma *"the young man and the young woman"*, constructed according to the syntactic rule G_E. We formalize a link *"and"* as intersection according to Minkovsky of fuzzy events $\tilde{x}$ and $\tilde{y}$: with eventological membership function $\mathbf{1}_{\tilde{x}(\cap)\tilde{y}}(\omega)$, the satisfying formula for the indicator of any set-operation above the set of M-fuzzy events $\tilde{X}$ (fig. 2).

In Zadeh's theory language links cannot be mathematically interpreted only by one type of conjunction in all situations. For example, the link "*and*" is presented by the t-norms, being special binary operations on an interval [0, 1]. The choice of a concrete kind of the formula for a link depends on the mutual attitude between fuzzy sets that leads, thus, to use of various t-norms [3]: a minimum T_M, probabilistic product T_P and Lukasiewicz's t-norm (fig. 2), etc. The basic eventological theorem of fuzzy events [4] offers the one and only general the eventologically correct formula for the indicator of any set-operation above any set of fuzzy events. Thus, the formula for eventological membership functions of E-event to result of the given set-operation is unique eventologically correct generalization of "usual" membership functions, various empirical variants used in set in the Zadeh's theory of fuzzy sets. The basic theorem has the general character as any set-operation habitual union, intersection and a symmetric difference can act, and also any other possible set-operation above set of fuzzy events. Because of absence of the similar theorem in the Zadeh's theory of fuzzy sets the set of variants of membership function to the same set-operations till now is used: actually, to each specific task the variant of membership function to this or that set-operation is searched. This essential lack managed to be avoided in the offered eventological theory of fuzzy events. Besides this the eventology specifies, that the structures of dependences of "usual" events of which fuzzy events consist, serve as the reason of plurality of variants of membership function in the Zadeh's theory of fuzzy sets. Only structures of dependences of events define a kind of eventological membership function. And a plurality of variants in the classical theory speaks that it is necessary to lean not only on membership functions, which do not bear the information on fuzzy events, and on all eventological distribution of set of events which make the given fuzzy event, and on eventological distribution of set of fuzzy events as it is suggested in new the eventological theory of fuzzy events.

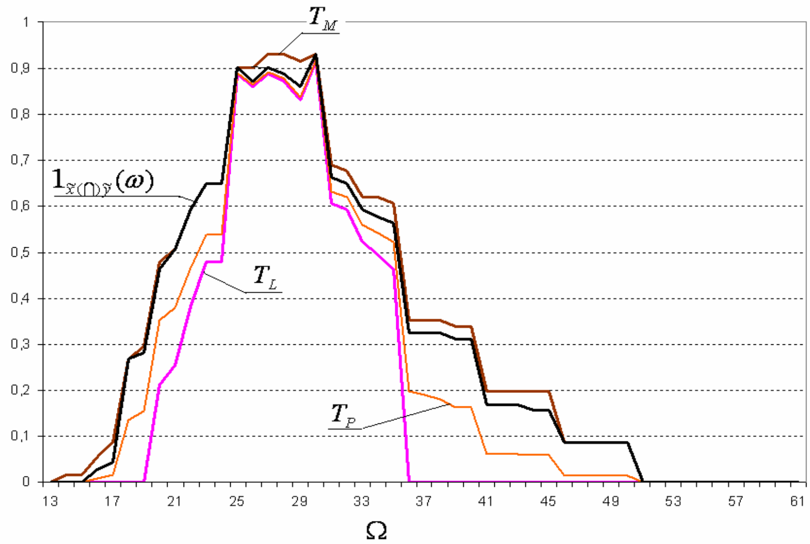

Fig. 2. Various interpretations of a syntagma " *the young man and the young woman*". The link *"and"* is presented by the indicator of intersection of two fuzzy events o according to Minkovsky $1_{\tilde{x}(\cap)\tilde{y}}(\omega)$ and the most popular t-norms [3]: a minimum T_M, probabilistic product and Lukasiewicz's t-norm.

The theory of fuzzy events is one of directions of the eventology [4], convincingly showing efficiency of eventological theory in knowledge of the phenomena and processes where the leaging role is played by the reasonable subject. The eventological theory of fuzzy events naturally mathematically proves and expands the Zadeh's theory of fuzzy sets as the approach in the mathematical description of uncertainty. In this work very simple example of eventological formalizations of a linguistic variable is considered. However this example allows to illustrate evidently the basic concepts and principles of the theory of fuzzy events.

Literature

1. L.A. Zadeh. Fuzzy sets. Inf. Control, 8. (1965) pp. 338-353.
2. L.A. Zadeh. The concept of linguistic variable and its application to approximate reasoning. American Elsevier Publishing Company (1973).
3. V. Novak, I. Perfilieva, J. Mochkorg. Mathematical principles of fuzzy logic. (In Russian) FIZMATLIT, Moskow, (2006) 352 p.
4. O.Yu. Vorob'ov. Eventology. (In Russian) SFU, Krasnoyarsk (2007) 435 p.